Program and abstracts of the workshop

Stein's method and stochastic geometry

September 18 - 20, 2024

Hamburg University of Technology

The workshop is supported by the DFG Priority Programme SPP 2265 Random Geometric Systems.

Organisers: Chinmoy Bhattacharjee and Matthias Schulte

Practical information

Venue

All talks will take place in **Room 0.16** of **Building H** on the campus of **Technische Universität Hamburg**, Am Schwarzenberg-Campus 5, 21073 Hamburg. For directions see the campus map.

The closest S-Bahn stations are Harburg Rathaus and Heimfeld, which are about 10 minutes by foot. One can also go by bus to Kasernenstraße (TU Hamburg). The closest station for long distance trains is Hamburg-Harburg. From there you can either take an S-Bahn to Harburg Rathaus or walk (about 25 minutes).

Conference dinner

All participants are invited to the conference dinner on Thursday evening at 18:30. The venue is the restaurant **Kaiserlich**, Schwarzenbergstraße 80, 21073 Hamburg, which is to the north of the campus on the other side of Schwarzenbergstraße.

Program

Wednesday, September 18

13:00 - 13:50	Registration and coffee break
13:50 - 14:00	Opening
14:00 - 14:55	Giovanni Peccati: *On the smoothness of nodal volumes*
15:00 - 15:55	Benedikt Rednoß: *Kolmogorov bounds and moderate deviations for functionals of Rademacher random variables*
15:55 - 16:30	Coffee break
16:30 - 17:25	Tara Trauthwein: *Multivariate Malliavin-Stein bounds under minimal moment assumptions*
17:30 - 17:50	Tillmann Bühler: *Intersection processes of k-flats in hyperbolic space: New limits and convergence rates for observations in spherical windows*

Thursday, September 19

9:30 - 10:25	Matthias Reitzner: *Binomial and Poisson random polytopes*
10:30 - 10:50	Leoni Carla Wirth: *Stein's method for spatial random graphs*
10:50 - 11:20	Coffee break
11:20 - 12:15	Daniel Hug: *Boolean models in hyperbolic space*
12:15 - 14:00	Lunch break
14:00 - 14:55	Christian Hirsch: *Limit theorems under heavy-tailed scenario in the age dependent random connection models*
15:00 - 15:20	Ercan Sönmez: *Applications of Stein's method via Palm coupling*
15:20 - 16:00	Coffee break
16:00 - 16:20	Matthias Lienau: *Large components in the subcritical random connection model*
16:25 - 16:45	Lianne de Jonge: *Limit theorems for projections of random geometric graphs*
16:50 - 17:10	Tiffany Lo: *On the rate of normal approximation for Poisson continuum percolation*
18:30 - 21:00	Conference dinner

Friday, September 20

9:30 - 10:25	Anna Gusakova: *Concentration inequalities for Poisson U-statistics with applications to stochastic geometry*
10:30 - 10:50	Martina Petráková: *About the furthest neighbour in Poisson-Laguerre tessellation with unbounded weights*
10:50 - 11:20	Coffee break
11:20 - 12:15	Christoph Thäle: *Random polytopes: recent results and future challenges*

Abstracts

On the smoothness of nodal volumes

Giovanni Peccati, University of Luxembourg

Wednesday 14:00 - 14:55

In the last decade, the "Malliavin-Stein method"has been successfully applied to the study of the high-energy (or large domain) fluctuations of local geometric functionals (like e.g. volumes of level sets) associated with Gaussian fields on manifolds. In most situations, the Malliavin-Stein method is applied through the use of Wiener chaos, that is: one first represents a given geometric object as an orthogonal sum of chaotic projections, and then one studies each projection by using such well-established tools as: isometric and product formulae for multiple integrals, hypercontractivity, fourth moment theorems, and so on. In the first part of my talk, I will argue that such an approach quickly becomes combinatorially/analytically unfeasible whenever one has to deal with fields defined on generic (and possibly high-dimensional) manifolds - so that it would be desirable to have access to such synthetic estimates as first- and second-order Poincaré inequalities, Nourdin-Viens formulae, reversed Berry-Esseen bounds, etc. In the second part of my talk, I will illustrate some first steps in this direction - contained in a recent joint work with M. Stecconi (2024) - in which we study the Malliavin-Sobolev regularity of nodal volumes of Gaussian fields in a general setting. Our results complete and extend a seminal work by J. Angst and G. Poly (2018) and cover the (previously unexplored) case of planar random functions. Along the way, if time permits, I will use ideas from recent works by Belyaev, McAuley and Muirhead (2021-23) in order to demonstrate that - for smooth stationary fields with integrable covariances - Gaussian fluctuations of nodal volumes originate from a stabilization mechanism *à la* Penrose-Yukich.

Kolmogorov bounds and moderate deviations for functionals of Rademacher random variables

Benedikt Rednoß, Ruhr-University Bochum

Wednesday 15:00 - 15:55

In this talk, we discuss normal approximation bounds for functionals over (possibly infinitely many) Rademacher random variables. First, a Kolmogorov bound is established by means of the discrete Malliavin–Stein method. This leads to a second-order Gaussian Poincaré inequality. Further, an intensive study of the behavior of operators from the Malliavin–Stein method along with the moment generating function of the Rademacher functional leads to moderate deviations for normal approximation of these functionals. As an application, the number of vertices with prescribed degree and the subgraph counting statistic in the Erdős–Rényi random graph are discussed.

Multivariate Malliavin-Stein bounds under minimal moment assumptions

Tara Trauthwein, University of Oxford

Wednesday 16:30 - 17:25

This talk will present new bounds for the normal approximation of multivariate Poisson functionals, achieved via the Malliavin-Stein method in combination with p-Poincaré inequalities. We will see examples for which such improved moment conditions are necessary, in a univariate and a multivariate context, and compare the use of Stein's method with an interpolation technique originating in the theory of spin systems.

Intersection processes of k-flats in hyperbolic space: New limits and convergence rates for observations in spherical windows

Tillmann Bühler, Karlsruhe Institute of Technology

Wednesday 17:30 - 17:50

Over the past years, there has been a lot of interest in the study of isometry invariant Poisson processes of k-planes in hyperbolic space $\mathbb{H}^d$ $(0 \leq k < d)$. A phenomenon that has no counterpart in euclidean geometry arises in the investigation of the total k-dimensional volume F_r of the process inside a spherical observation window B_r, when one lets r tend to infinity: While F_r is asymptotically normally distributed for $2k \leq d+1$, it has been shown to obey a nonstandard central limit theorem for $2k > d+1$.

The intersection process of order m (for $d - m(d-k) \geq 0$) of the original process η consists of all intersections of distinct planes $E_1, \ldots, E_m \in \eta$ with $\dim(E_1 \cap \cdots \cap E_m) = d - m(d-k)$. For this process, the total $d - m(d-k)$-dimensional volume $F_r^{(m)}$ of the process in B_r is of particular interest. For $2k \leq d+1$ it has been shown that $F_r^{(m)}$ is again asymptotically normally distributed. For $m \geq 2$, the limit is so far unknown, although it has been shown for certain d and k that it cannot be a normal distribution.

We determine the limit distribution for all values of d, k, m. We also give explicit rates of convergence in the Kolmogorov distance.

This is joint work with Daniel Hug.

Binomial and Poisson random polytopes

Matthias Reitzner, University of Osnabrück

Thursday 9:30 - 10:25

Choose n uniform iid random points in a convex set $K \subset \mathbb{R}^d$. The convex hull of these random points is the binomial random polytope. Of interest are combinatorial quantities like the number of vertices, edges and faces, and metric quantities like the volume or, more general, the intrinsic volumes. There are well known methods to investigate the expectations of these quantities.

Yet to derive results for variances or distributional results it turns out to be convenient to introduce a further randomization. Assuming the number of random points to be Poisson distributed instead of precisely n, defines the Poisson random polytope.

We describe the advantages, problems and limitations of switching between these two models by Poissonizing and de-Poissonizing the results for random polytopes.

Stein's method for spatial random graphs

Leoni Carla Wirth, University of Göttingen

Thursday 10:30 - 10:50

Spatial random graphs provide an important framework for the analysis of relations and interactions in networks. In particular, the random geometric graph has been intensively studied and applied in various frameworks like network modeling or percolation theory.

In this talk we focus on approximation results for a generalization of the random geometric graph that consists of vertices given by a Gibbs process and (conditionally) independent edges generated from a connection probability function. We introduce a new graph metric between finite spatial graphs of possibly different sizes that is built on the OSPA metric for point patterns, but penalizes both vertex and edge structures. We develop Stein's method for the Wasserstein distance with respect to this graph metric and obtain general rates of convergence for a suitable type of convergence in distribution of spatial random graphs. Finally, we present an application of our approximation results to the percolation graph of large balls in a Boolean model.

Joint work with Dominic Schuhmacher (University of Göttingen).

Boolean models in hyperbolic space

Daniel Hug, Karlsruhe Institute of Technology

Thursday 11:20 - 12:15

The union of the particles of a stationary Poisson process of compact (convex) sets in Euclidean space is called Boolean model and is a classical topic of stochastic geometry. We consider Boolean models in hyperbolic space, where one takes the union of the particles of a stationary Poisson process in the space of compact (convex) subsets of the hyperbolic space. Geometric functionals such as the volume (or some other intrinsic volume) of the intersection of the Boolean model with a compact convex observation window are studied. In particular, the asymptotic behavior for balls with increasing radii as observation windows is investigated. Exact and asymptotic formulas for expectation, variances and covariances are shown and univariate and multivariate central limit theorems are derived. Compared to the Euclidean framework, some new phenomena can be observed. (Based on joint work with Günter Last and Matthias Schulte)

Limit theorems under heavy-tailed scenario in the age dependent random connection models

Christian Hirsch, Aarhus University

Thursday 14:00 - 14:55

This talk considers the limit theorems associated with the subgraph counts in the age-dependent random connection model. More precisely, first, under suitable assumptions on the tree shape, we identify regimes where the subtree count converges to a stable random variable. As an intermediate result, we obtain convergence of an associated point process towards a Poisson point process. Finally, we prove the same type of results for the clique counts. Here, a crucial ingredient is the derivation of the expectation asymptotics for cliques, which is a result of independent interest.

This talk is based on joint work with T. Owada.

Applications of Stein's method via Palm coupling

Ercan Sönmez, Ruhr-University Bochum

Thursday 15:00 - 15:20

We explore general Poisson approximations connected to Stein's method through Palm coupling, with a focus on applications in the random connection model based on a marked Poisson point process. Specifically, we examine the distribution of edge lengths in different versions of this model, identifying phase transitions. This presentation is based on joint work with Arnaud Rousselle and Clara Stegehuis.

Large components in the subcritical random connection model

Matthias Lienau, Hamburg University of Technology

Thursday 16:00 - 16:20

The random connection model is an inhomogeneous random graph whose vertex set is given by a stationary Poisson process in $\mathbb{R}^d$. All vertices are equipped with positive i.i.d. weights. Any two vertices get connected via an edge with a probability which is increasing in the weights of the endpoints but decreasing in their distance. The resulting graph exhibits natural clustering effects as well as a scale-free behaviour for a suitable weight distribution. For our results we require the graph to have sufficiently few edges. We study the point process of component sizes in a growing observation window and derive, under suitable rescaling, weak convergence to a Poisson process. This implies that the rescaled size of the largest component in the observation window converges to a Fréchet distribution.

This talk is based on joint work with Matthias Schulte.

Limit theorems for projections of random geometric graphs

Lianne de Jonge, University of Osnabrück

Thursday 16:25 - 16:45

Let G be a random geometric graph where the vertices are determined by a Poisson point process on some compact convex set $W \subset \mathbb{R}^d$. We consider the edge crossings in a random graph drawing generated by projecting G onto a plane. These crossings define a point process on the plane. We show that for sparse G the point process converges to a Poisson point process as the vertex intensity goes to infinity. The number of crossings and a second quantity called the stress satisfy a multivariate central limit theorem in the thermodynamic regime.

This talk is based on joint work with Hanna Döring.

On the rate of normal approximation for Poisson continuum percolation

Tiffany Lo, Stockholm University

Thursday 16:50 - 17:10

It is shown in Penrose (2003) that the cardinality of the largest cluster of a percolating Poisson process restricted to a large finite box is asymptotically normal. In this talk, I will explain how we can use Stein's method to complement this result with a convergence rate. As each point in the largest cluster is determined by points as far as the diameter of the box, we use the fact that the second largest cluster has comparatively shorter range of dependence to restrict the range of dependence. Then, apply a recent result of Chen, Röllin and Xia, (2021) to obtain a Berry-Esseen type bound for the normal approximation of the number of points belonging to clusters that have a restricted range of dependence, and finally estimate the gap between this quantity and the cardinality of the largest cluster.

Joint work with Aihua Xia.

Concentration inequalities for Poisson U-statistics with applications to stochastic geometry

Anna Gusakova, University of Münster

Friday 9:30 - 10:25

Let η be a Poisson point process on a general measurable space. A Poisson functional is a random variable $F(\eta)$, such that almost surely we have $F(\eta) = f(\eta)$ for some measurable veal valued function f on the space of counting measures. Poisson functionals have been intensively studied within last years and they play an important role in stochastic geometry since many important geometric functionals of stochastic geometry models are in fact Poisson functionals. Poisson U-statistic is an example of Poisson functional, which has particularly nice structure. In this talk we present concentration inequalities for Poisson U-statistics under some rather mild conditions. We will discuss their optimality and consider a few applications to stochastic geometry models.

Based on a joint work with Gilles Bonnet.

About the furthest neighbour in Poisson-Laguerre tessellation with unbounded weights

Martina Petráková, Charles University

Friday 10:30 - 10:50

Abstract: The object of our research is the Poisson-Laguerre tessellation, i.e. a random Laguerre tessellation whose generator is a stationary Poisson marked point process. In order to use Malliavin-Stein method to study asymptotic behaviour of functionals of the Poisson-Laguerre tessellation, it is necessary to control the distance to the furthest neighbour of a cell generated by an added point. In this talk, we will focus on the case, where the weights of the random generator are not uniformly bounded, and present some properties of the distance to the furthest neighbour, which were derived using the concept of tempered configurations.

This talk is based on joint work with Zbyněk Pawlas.

Random polytopes: recent results and future challenges

Christoph Thäle, Ruhr-University Bochum

Friday 11:20 - 12:15

In this talk I will describe selected results on the geometry of random polytopes that have been developed during the last decade. Special attention will be paid to fluctuation results obtained with - or without - Stein's method. In this context I will also outline a number of open problems that might be regarded as future challenges in this area.