

INFINITE TREES OVER NON-ARCHIMEDEAN ORDERED FIELDS

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An ordered field is a field together with a total ordering (\succ) of its elements that is compatible with the field operations (see [4, p. 449]). An ordered field \mathbb{K} is *non-Archimedean* if there exist an *infinitesimal*, i.e. $\epsilon \in \mathbb{K}$ such that

$$\epsilon \prec \frac{1}{n} = \frac{1}{\underbrace{1 + \dots + 1}_{n \text{ times}}}$$

for any $n \in \mathbb{N} \subset \mathbb{K}$.

One very important example of non-Archimedean field is the Levi-Civita field, which is described in e.g. [2].

Instead of usual real weights we consider weights from non-Archimedean field \mathbb{K} on infinite locally finite trees. Since the monotone convergence theorem does not hold in non-Archimedean ordered fields, the property of graph to be either recurrent or transient fails, which is pointed out in [5].

The goal of this project is to investigate behaviour of infinite locally finite trees using the energy of the solutions of the Dirichlet problems on finite approximations (see [3], [5]), starting with spherically symmetric trees, described in [1].

This project is suited for 3 to 4 students.

REFERENCES

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- [4] Serge Lang. *Algebra*. Springer, 2002.
- [5] Anna Muranova. *Effective impedance over ordered fields*. Journal of mathematical physics 62, 033502(2021)