A PROBABILISTIC PERPECTIVE ON RECURRENCE AND TRANSIENCE

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In the ISem lectures, the concepts of recurrence and transience for a graph over X were defined analytically via the spaces $\mathcal{D}(X)$ and $\mathcal{D}_0(X)$. The aim of this project is to understand the probabilistic meaning of these notions: recurrence means that the Markov process associated with the graph returns infinitely often to any given vertex, whereas transience means that eventually the process will never return again.
There are actually two ways to associate a stochastic process with a graph: a time discrete version that corresponds to a random walk on the graph, and a time continuous version that is related to the semigroup $(e^{-tL})_{t\geqslant 0}$ and kernels p_t from the ISem lectures. The basic idea is that $p_t(x,y)$ is related to the probability that a particle starting at the vertex x at time 0 will be at the vertex y at time t. Background material on discrete- and continous-time Markov chains can be found in [2].
Starting point of the project is the book [1] on which the ISem lectures are based. In Sections 0.10 and 2.5 one can find the definition of the process associated to a graph. Then Section 6.6 deals with the probabilistic view on recurrence; in particular, in Theorem 6.35 the equivalence of the analytic and probabilistic descriptions of recurrence is proved. An interesting 'side quest' can be found in Section 7.9, where a probabilistic interpretation of stocastic completeness is given. The precise selection of topics will be decided among the participants of the project.

This project is suited for 3 to 4 students.

References

[1] M. Keller, D. Lenz, R. Wojciechowski, *Graphs and Discrete Dirichlet Spaces.* Grundlehren der mathematischen Wissenschaften 358, Springer, Cham, 2021.
`https://www.math.uni-potsdam.de/fileadmin/user_upload/Prof-GraphTh/Keller/KellerLenzWojciechowski_GraphsAndDiscreteDirichletSpaces_personal.pdf`

[2] J. R. Norris, *Markov Chains.* Cambridge Series in Statistical and Probabilistic Mathematics, Cambridge University Press, 1997.