GENERAL DIRICHLET FORM THEORY

SASCHA TROSTORFF AND MARCUS WAURICK

In this year's ISem [1] we have encountered Dirichlet Forms in Lecture 7 and have applied the Beurling–Deny Theorems to the concrete form arising from a graph. It is the purpose of this project to provide more examples for Dirichlet forms and to study the associated Markov semigroups.
One main goal of the project is to prove the following representation result due to Beurling and Deny:

Theorem ([2, Theorem 3.2.1])**.** *Let $\mathcal{E}$ be a regular Dirichlet form on $L_2(X,m)$. Then there exist uniquely determined Radon measures σ and k on $X\times X\setminus\Delta$ and X respectively, such that*

$$\mathcal{E}(u,v)=\mathcal{E}^{(c)}(u,v)+\int_{X\times X\setminus\Delta}(u(x)-u(y))(v(x)-v(y))\,\mathrm{d}\sigma(x,y)+\int_X u(x)v(x)\,\mathrm{d}k(x)$$

for each $u,v\in C_c(X)\cap\mathrm{dom}(\mathcal{E})$. Here, Δ is the diagonal of $X\times X$ and $\mathcal{E}^{(c)}$ is a strongly local symmetric form.

Moreover, if time permits, we study the concept of recurrence and transience for general Dirichlet forms and their associated Markov semigroups and compare them with the results presented in the ISem.

This project is suited for 3 to 4 students.

References

[1] ISem 26, Lecture Notes, 2023. `https://www.mat.tuhh.de/veranstaltungen/isem26/_media/lecturenotes.pdf`
[2] M. Fukushima, Y. Oshima, M. Takeda, Dirichlet Forms and Symmetric Markov Processes, 2nd Edition, De Gruyter, Berlin/New York, 2011