LAPLACIANS ON INFINITE GRAPHS: FROM CONTINUOUS TO DISCRETE AND BACK

NOEMA NICOLUSSI

There are two main notions of Laplacian operators associated with infinite graphs: in the first setting, graphs are treated as discrete objects and discrete Laplacians are defined by difference expressions. In the second setting, one turns the graph into a continuous geometric object by replacing edges with intervals and glueing them together at vertices. This geometric object is called a metric graph and allows to consider Laplacian differential operators on graphs (often called quantum graphs).

In this project, we will study the relationship between these two settings. A systematic connection has been established recently in [3, 4]. To each Laplacian on a metric graph, one can associate a discrete Laplacian which has similar spectral and parabolic properties. Moreover, all discrete Laplacians on locally finite graphs arise in this way. This leads to a systematic way of connecting the settings and several applications.

We first introduce metric graphs and their Laplacians [1, 3]. Following [4], we then study connections between Laplacians on discrete and metric graphs on different levels. Here the participants can choose topics which are interesting to them, e.g.,

- Intrinsic metrics and quasi-isometric spaces
- Spectral estimates, isoperimetric constants and Cheeger's inequality
- Stochastic completeness and recurrence/transience
- Ultracontractivity estimates for the heat semigroup

For finding out whether this project is interesting to them, the participants can also consult [5, Section 5], which contains a short overview of these thematics.

This project is suited for 3 to 4 students.

References

[1] G. Berkolaiko and P. Kuchment, *Introduction to Quantum Graphs*, Amer. Math. Soc., Providence, RI, 2013.

[2] ISem 26, Lecture Notes, 2023. `https://www.mat.tuhh.de/veranstaltungen/isem26/_media/lecturenotes.pdf`

[3] A. Kostenko and N. Nicolussi, *Laplacians on infinite graphs: discrete vs. continuous*, Proceedings of the 8ECM, to appear, arXiv:2110.03566 (2021).

[4] A. Kostenko and N. Nicolussi, *Laplacians on infinite graphs*, Mem. Eur. Math. Soc., to appear; preprint available at `www.mat.univie.ac.at/~kostenko/list/GraphLaplInf.pdf`.

[5] N. Nicolussi, *A note on Spectral Analysis of Quantum graphs*, arXiv:2209.02968 (2022).