SPECTRA OF PERIODIC QUANTUM GRAPHS

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This project builds on recent results on discrete Laplacians [1] on *periodic graphs* which emerge for instance in physics when modelling two-dimensional materials. The eventual goal is to generalize these results to so-called *quantum graphs* and continuous Schrödinger operators defined on them. We will start with discrete graphs with a two-dimensional symmetry as investigated in [4, 3] and introduce – as a main tool – Floquet theory which allows to reduce periodic operators on infinite graphs to operators with a compact resolvent. In a next step, we will learn about *metric graphs* as well as Schrödinger operators defined on them. Focussing on the Laplacian, we want to understand the correspondence between the spectrum of quantum and the corresponding discrete graphs which holds in certain cases [2]. Finally, if time permits, we shall see whether the results regarding the spectrum of discrete Laplacians as obtained in [3] can be translated to the setting of quantum graphs.

This project is suitable for 3–4 students.

References

[1] ISem 26, Lecture Notes, 2023. `https://www.mat.tuhh.de/veranstaltungen/isem26/_media/lecturenotes.pdf`

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[3] J. Wintermayr, J. Kerner and M. Täufer. Robustness of flat bands on the perturbed Kagome and the perturbed Super-Kagome lattice. arXiv:2301.05076 [math.SP], 2023.

[4] N. Peyerimhoff and M. Täufer. Eigenfunctions and the integrated density of states on Archimedean tilings. arXiv:1807.06406 [math-ph]. *J. Spectr. Theory*, 11(2):461–488, 2021.