

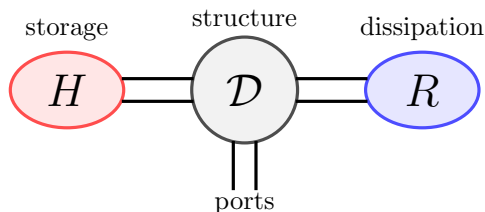
PORT-HAMILTONIAN SYSTEMS ON GRAPHS

RENÉ HOSFELD AND MERLIN SCHMITZ

Port-Hamiltonian systems (pHS) theory is gaining more and more attention in the last three decades. They are a powerful tool to model and analyze a broad class of energy based physical systems, including multi-body systems, electrical circuits and electromechanical systems. In a simple, finite-dimensional case pHS are of the form

$$\begin{aligned}\dot{x}(t) &= (J - R) \frac{\partial H}{\partial x}(x(t)) + Bu(t), \\ y(t) &= B^T \frac{\partial H}{\partial x}(x(t)) + Du(t),\end{aligned}$$

where the *Hamiltonian* $H: \mathbb{R}^n \rightarrow \mathbb{R}$ represents the energy of the system, $J = -J^T \in \mathbb{R}^{n \times n}$ the internal interconnection, $0 \leq R \in \mathbb{R}^{n \times n}$ describes dissipation of energy and $0 \leq D \in \mathbb{R}^{m \times m}$ is a feedthrough. The input and collocated output functions u, y , taking values in \mathbb{R}^m , are called the *ports* that connect the system to the environment or other pHS via a matrix $B \in \mathbb{R}^{n \times m}$. One characteristic of such a system is its modular structure – it can be separated into the three building blocks of energy storage, dissipation and the geometrical interconnection which is contained in the so called *Dirac structure*.



Given a (finite) directed graph with a set of boundary vertices, we investigate different Dirac structures respecting the graph's geometry. Following [1], we derive corresponding pHS. The abstract theory is illustrated by means of mass-spring-damper systems. Necessary basics on pHS are given in [2].

Depending on the students background we investigate further analytic properties.

This project is suited for 3 to 4 students.

REFERENCES

- [1] Arjan van der Schaft and Bernhard Maschke. Port-Hamiltonian systems on graphs. *SIAM Journal on Control and Optimization*. 51:906–937 (2013). <https://epubs.siam.org/doi/pdf/10.1137/110840091>
- [2] Arjan van der Schaft and Dimitri Jeltsema. *Port-Hamiltonian Systems Theory: An Introductory Overview*. Foundations and Trends[®] in Systems and Control, vol. 1, no. 2-3, pp. 173–378, 2014. <https://www.math.rug.nl/~arjan/DownloadVarious/PHbook.pdf>