STABILITY OF THE WAVE EQUATIONS ON NETWORKS

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Networks have been studied widely in recent years with motivations from and applications to classical natural sciences (electro-circuits, chemical processes, neural networks, population biology, etc.) as well as to social sciences or even to the WorldWideWeb. Much progress has been made in understanding the structure of these networks, and we refer to M.E.J. Newman [7] for a survey on these developments. However, on p. 224 of [7] he says: "The next logical step after developing models of network structure, $(\cdots)$ is to look at the behavior of models of physical (or biological or social) processes going on on those networks. Progress on this front has been slower than progress on understanding network structure."

The main goal of the present project is to define an appropriate setting and to find the tools to investigate such processes on networks. Here we combine functional analytical and graph theoretical methods in order to study wave equations in networks. We show that these equations can be described by a strongly continuous operator semigroup on a Hilbert space. Using frequency domain analysis we prove that the semigroup behaves asymptotically exponential/polynomial. These results have been already shown for a single elastic string with local Kelvin–Voigt damping, see [5, 6].

The first part of the project will therefore deal with defining the proper functional setting for wave equations on networks, see [1] as well as showing that the resulting system is well-posed, see [3, 4, 2].

Afterwards, and depending on the interests of the participants, we will deal with the asymptotic behavior of the corresponding semigroup defined on the network.

This project is suited for 3 to 4 students.

References

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