POSITIVITY PRESERVING AND IMPROVING C_0-SEMIGROUPS

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For Dirichlet Laplacians L on graphs, we have seen in [1, Theorem 6.1] that the corresponding semigroup operators e^{-tL} are positivity improving for each $t > 0$ if and only if the resolvent operators $(L+\alpha)^{-1}$ are positivity improving for all $\alpha > 0$. More generally speaking, the same equivalence holds for each positivity preserving semigroup on a suitable ordered space. In this project, we look at a proof for the case of L^p-spaces.

In addition, we study positivity preserving *irreducible* semigroups. In particular, we will see that a positivity preserving irreducible analytic semigroup is also positivity improving. The main source of references will be [2, 3, 4].

This project is suited for 3 to 4 students.

References

[1] ISem 26, Lecture Notes, 2023. `https://www.mat.tuhh.de/veranstaltungen/isem26/_media/lecturenotes.pdf`

[2] A. Bátkai, M. Kramar Fijavž, & A. Rhandi. Positive operator semigroups. From finite to infinite dimensions. *Oper. Theory: Adv. Appl.*, **257** pp. xviii + 364 (2017).

[3] W. Arendt, R. Nagel, A. Grabosch, G. Greiner, U. Groh, H.P. Lotz, U. Moustakas, F. Neubrander, & U. Schlotterbeck. One-parameter semigroups of positive operators. Lecture Notes in Mathematics, vol. 1184, Springer-Verlag Berlin Heidelberg, 1986.

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