Nonlinear Evolutionary Equations

Picard's Theorem And Applications

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Outline

Section 1

Introduction

Want to study **Nonlinear Evolutionary Inclusions** of the type

$$\partial_{t,\nu} M(\partial_{t,\nu}) + A \ni (u, f)$$

Assumptions:

- $M : \mathrm{dom}(M) \subseteq \mathbb{C} \to \mathrm{L}(H)$ is a material law satisfying the coercivity condition

$$\mathrm{Re}\,\langle zM(z)\phi, \phi\rangle \geqslant c\,\|\phi\|^2$$

  i.e. $zM(z) - c$ is monotone (as we will see)
- $A \subseteq L_{2,\nu}(\mathbb{R}, H) \times L_{2,\nu}(\mathbb{R}, H)$ is maximal monotone and autonomous

$\to$ *Want analogue to* **Picard's Theorem**

Subsection 1

(Maximal) Monotonicity

Recall: Relations

A **relation** on a real Hilbert space H is a subset of $H \times H$. For any relations A, B we define the

scaled relation $\lambda A := \{(x, \lambda y) | (x, y) \in A\}$,

inverse relation $A^{-1} := \{(y, x) | (x, y) \in A\}$,

sum $A + B := \{(x, y + z) | x, y, z \in H \text{ s.t. } (x, y) \in A \wedge (x, z) \in B\}$,

domain $\mathrm{dom}(A) = \{x | (x, y) \in A\}$ and

range $\mathrm{ran}(A) = \{y | (x, y) \in A\}$

A relation is said to be

surjective if $\mathrm{ran}(A) = H$,

injective if for every $y \in \mathrm{ran}(A)$, $A^{-1}(\{y\})$ is a singleton,

an **operator** if for every $x \in \mathrm{dom}(A)$, $A(\{x\})$ is a singleton.

Maximal Monotonicity

For this part of the talk, let H be a real Hilbert space.

(See complex H as real Hilbert space $H_{\mathbb{R}}$: $\langle x\,,\,y\rangle_{H_{\mathbb{R}}} = \mathrm{Re}\,\langle x\,,\,y\rangle_{H}$)

We call a relation $A \in H \times H$:

- **monotone** if

$$\forall (x, y), (x', y') \in A : \langle y' - y\,,\,x' - x\rangle \geqslant 0$$

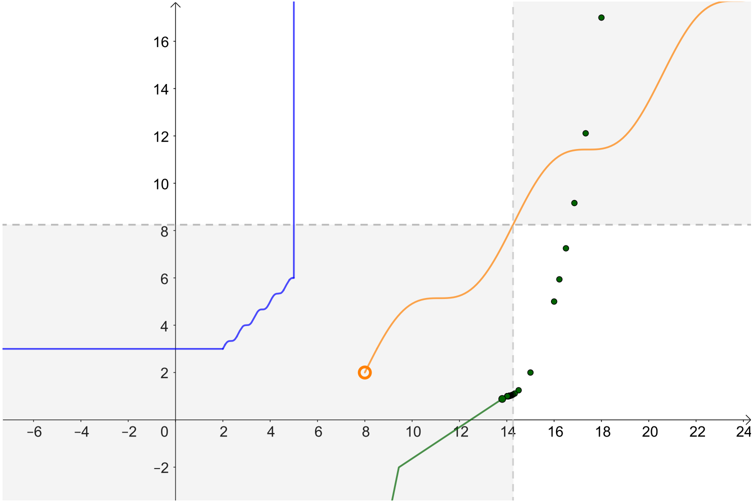
- **maximal monotone** if it is maximal w.r.t. inclusion among monotone relations.

Linear (max.) monotone operators are called **(m-)accretive**.

$$\forall x \in \mathrm{dom}(A) : \langle Ax\,,\,x\rangle \geqslant 0$$

Monotone Relations in $\mathbb{R} \times \mathbb{R}$

A relation $A \in \mathbb{R} \times \mathbb{R}$ is **monotone** if $\forall (x, y), (x', y') \in A : (y' - y)(x' - x) \geqslant 0$.



$A \in H \times H$ is **monotone** if $\forall (x, y), (x', y') \in A : \langle y' - y\,, x' - x\rangle \geqslant 0.$

Proposition

Let H be a Hilbert space and A be a relation in $H \times H$ s.t. $A - c$ is monotone for some $c > 0$. Then

$$\|u - v\| \leqslant \frac{1}{c}\|f - g\| \qquad \forall (u, f), (v, g) \in A$$

In particular, A is injective and A^{-1} is a Lipschitz operator.

Proof.

Let $(u, f), (v, g) \in A$. Then $(u, f - cu), (v, g - cv) \in A - c$ and using the monotonocity of $A - c$, we obtain

$$\langle (f - cu) - (g - cv)\,, u - v\rangle \geqslant 0.$$

Hence

$$\langle f - g\,, u - v\rangle \geqslant c\|u - v\|^2.$$

The result now follows by Cauchy-Schwarz-inequality. □

Proposition

Let H be a Hilbert space and A be a relation in $H \times H$ s.t. $A - c$ is monotone for some $c > 0$. Then

$$\|u - v\| \leqslant \frac{1}{c}\|f - g\| \qquad \forall (u, f), (v, g) \in A$$

In particular, A is injective and A^{-1} is a Lipschitz operator.

Definition

Let A be a relation in a real Hilbert space H. If $A - c$ is monotone for some $c > 0$, we call A **strictly monotone**.

Lemma

Let A be monotone and (x_n, y_n) a sequence of points in A such that either

$x_n \to x$ *strongly and* $y_n \rightharpoonup y$ *weakly, or*

$y_n \to y$ *strongly and* $x_n \rightharpoonup x$ *weakly.*

Then $A \cup \{(x, y)\}$ is monotone. In particular, if A is maximal monotone, then it is necessarily closed in $H \times H$.

Sketch of the Proof:
Let $(u, f) \in A$. Then

$$0 \leqslant \langle x_n - u \, , \, y_n - f \rangle \to \langle x - u \, , \, y - f \rangle$$

Let A a skew-selfadjoint operator on H. Then for all $x \in \operatorname{dom}(A)$ we have

$$\langle Ax, x \rangle = -\langle x, Ax \rangle$$

which implies that $\langle Ax, x \rangle = 0 \geqslant 0$.
Thus A is monotone (i.e. accretive).

Examples for Monotone Relations

Let H be a real Hilbert space and $M : \mathrm{dom}(M) \subseteq H \to H$ a linear operator that satisfies the coercivity condition

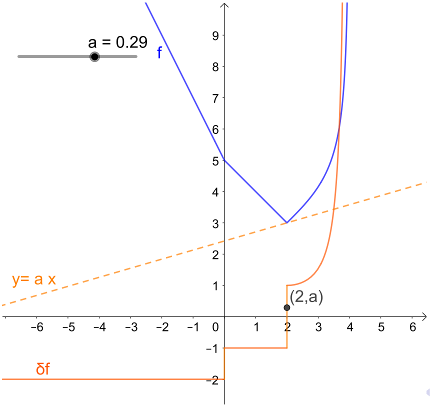
$$\langle x, Mx \rangle \geqslant c\|x\|^2 \qquad \forall x \in \mathrm{dom}(M)$$

for some $c > 0$, then $M - c$ is monotone and thus M is strictly monotone.

Rockafellar's Theorem A

$f : \Omega \subseteq \mathbb{R}^n \to \mathbb{R}$ differentiable convex, then gradient ∇f is monotone

More general:
$f : H \to \mathbb{R} \cup \{+\infty\}$ convex and lower semi-continuous, then the subdifferential $\partial f \subseteq H^2$ is **maximal** monotone



Subsection 2

Minty's Theorem

Minty's Theorem

Theorem (Minty's Theorem)

Let H be a Hilbert space and A be a monotone relation in $H \times H$. Then A is maximal monotone if and only if $1 + \lambda A$ is surjective for some/all $\lambda > 0$.

For proof: Wlog $\lambda = 1$ (A mon. $\iff$ λA mon.)

Proof of $1 + A$ surjective $\implies$ A : max. mon.

Recall: Strict mon. $\implies$ injective

Let B : mon. extension of A.

$\implies$ $1 + B$: injective extension of $1 + A$ (surjective)

$\implies$ $B = A$.

Step 1: Claim: Suffices to show $0 \in \operatorname{ran}(1 + A)$
Let $f \in H$, then

$$A - (0, f) \text{ is max. mon.} \overset{\text{Claim}}{\implies} 0 \in \operatorname{ran}(1 + A - (0, f)) \implies f \in \operatorname{ran}(1 + A).$$

Step 2: Proof of claim
Let $(u, f) \in H \times H$. Since A is max. mon.

$$\langle u - v\,, f - g\rangle \geqslant 0\,\forall\,(v, g) \in A \implies (u, f) \in A.$$

and so

$$\inf\{\langle u - v\,, f - g\rangle : (v, g) \in A\} \leqslant 0.$$

Consider **Fitzpatrick function** $F_A : H \times H \to -(\infty, \infty]$

$$F_A(u,f) = - \inf_{(v,g)\in A} \left(\langle u - v \,,\, f - g \rangle - \langle u \,,\, f \rangle \right)$$
$$= \sup_{(v,g)\in A} \left(\langle u \,,\, g \rangle + \langle v \,,\, f \rangle - \langle v \,,\, g \rangle \right)$$

Then (i) $F_A(u,f) \geqslant \langle u \,,\, f \rangle$
(ii) $F_A(u,f) = \langle u \,,\, f \rangle \iff (u,f) \in A$

One can show that $\exists\, (u_0, f_0)$ s.t.

$$0 \geqslant F_A(u_0, f_0) + \|u_0\|^2 + \|f_0\|^2 + F_A(-f_0, -u_0) \overset{(i)}{\geqslant} \|u_0 + f_0\|^2$$

It follows

$$F_A(u_0, -u_0) = \langle u_0 \,,\, -u_0 \rangle \overset{(ii)}{\Longrightarrow} (u_0, -u_0) \in A$$
$$\Longrightarrow 0 \in \operatorname{ran}(1 + A).$$

Corollary

Let A be a strictly monotone relation on a real Hilbert space with dense range. Then $\overline{A}$ is maximal monotone.

Yosida approximation

Let A be max. mon. and $\lambda > 0$. Define

$$A_\lambda := \lambda^{-1}\left(1 - (1+\lambda A)^{-1}\right)$$

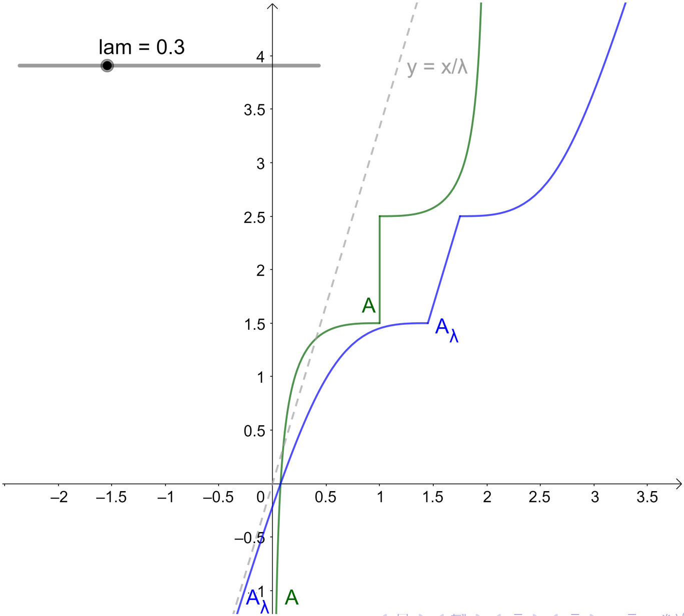
to be the **Yosida-approximation of A at λ**.

$$\text{Minty's theorem} \implies \operatorname{dom}(A_\lambda) = H.$$

Properties:

- A_λ is mon.
- A_λ is Lipschitz continuous and $\|A_\lambda\|_{\text{Lip}} \leqslant \frac{1}{\lambda}$.

Yosida approximation



Proposition

Let A, B : *maximal monotone,* $\lambda > 0$ *and* $y \in H$.

- *If* B *is Lipschitz continuous, then* $A + B$ *is maximal monotone.*
- *Let* $c > 0$ *and* $x_\lambda = (c + A + B_\lambda)^{-1} y$. *If*

$$C(y) := \limsup_{\lambda \to 0} \|B_\lambda x_\lambda\| < \infty,$$

then $\exists\, x \in H$ *s.t.* $x_\lambda \to x$ *and* $(x, y) \in c + A + B$.

In particular, $A + B$ *is maximal monotone if* $C(y) < \infty$ *for every* $y \in H$.

Section 2

Solution Theory for Nonlinear Evolutionary Inclusions

Subsection 1

Time Derivative

Recap

Let $\nu \neq 0$. Then $I_\nu : L_{2,\nu}(\mathbb{R}, H) \to L_{2,\nu}(\mathbb{R}, H)$ is defined as

$$I_\nu = \begin{cases} \mathbb{1}_{[0,\infty)} *, & \nu > 0 \\ -\mathbb{1}_{(-\infty,0]} *, & \nu < 0 \end{cases}$$

and

$$\partial_{t,\nu} := I_\nu^{-1}.$$

$M : \operatorname{dom} M \subseteq \mathbb{C} \to \mathcal{L}(H)$ is called a **material law** if $\operatorname{dom} M$ is open, M is holomorphic and $\exists\, \nu \in \mathbb{R}$ s.t. $\mathbb{C}_{\mathrm{Re}>\nu} \subseteq \operatorname{dom} M$ and

$$\sup_{z \in \mathbb{C}_{\mathrm{Re}>\nu}} \|M(z)\| < \infty. \qquad (*)$$

Set

$$s_b(M) := \inf\{\nu \in \mathbb{R} : (*) \text{ holds}\}$$

Theorem

Let M be a material law, $\nu_0 > s_b(M)$ and $c > 0$ be s.t. $zM(z) - c$ is monotone whenever $\operatorname{Re} z \geqslant \nu_0$. Then $\partial_{t,\nu} M(\partial_{t,\nu})$ is a strictly **maximal** *monotone operator on $L_{2,\nu}(\mathbb{R}, H)$ for all $\nu \geqslant \nu_0$.*

Proof

Since $\partial_{t,\nu} M(\partial_{t,\nu})$ is unitarily equivalent to $(i\mathbf{m}+\nu)M(i\mathbf{m}+\nu)$,

Enough to show: $A := (i\mathbf{m}+\nu)M(i\mathbf{m}+\nu)$ is strictly max. mon.

$$\begin{aligned} zM(z)-c \text{ mon.} &\implies A \text{ and } A^* \text{ are strictly mon.} \qquad (*) \\ &\implies A^* \text{ is injective} \\ &\implies A \text{ has dense range} \\ &\underset{\text{Minty}}{\overset{\text{Cor.}}{\implies}} A \text{ is max. mon.} \\ &\overset{(*)}{\implies} A \text{ is strictly max. mon.} \end{aligned}$$

Translation Operator:

$\tau_h : L_{2,\nu}(\mathbb{R}, H) \to L_{2,\nu}(\mathbb{R}, H),\ f(t) \mapsto f(t+h)$

$\to$ linear, invertible, bounded with $\|\tau_h\| = e^{\nu h}$ and adjoint $\tau_h^* = e^{2\nu h}\tau_{-h}$

$A \subseteq L_{2,\nu} \times L_{2,\nu}$ **autonomous** $:\Longleftrightarrow \forall (u,f) \in A : (\tau_h u, \tau_h f) \in A$

$\nu > s_b(M)$, then $\partial_{t,\nu}$ and $M(\partial_{t,\nu})$ are autonomous

Lemma

Let $f \in L_{2,\nu}(\mathbb{R}, H)$*, then:*

$$f \in \mathrm{dom}(\partial_{t,\nu}) \iff \left(\frac{\tau_h f - f}{h}\right)_{0<h<1} \text{ is bounded}$$

$$\iff \frac{\tau_h f - f}{h} \text{ conv. strongly as } h \to 0 \text{ (to } \partial_{t,\nu} f)$$

For instance,

$$\left\| \frac{\tau_h f - f}{h} \right\|_{2,\nu} = \left\| \frac{1}{h} \int_t^{t+h} \partial_{t,\nu} f(s) \, \mathrm{d}s \right\|_{2,\nu} = \left\| \tfrac{1}{h} \mathbb{1}_{[-h,0]} * \partial_{t,\nu} f \right\|_{2,\nu}$$

$$\leqslant \underbrace{\left\| \tfrac{1}{h} \mathbb{1}_{[-h,0]} \right\|_{1,\nu}}_{= \frac{e^{h\nu}-1}{h\nu} = 1 + \mathcal{O}(h)} \left\| \partial_{t,\nu} f \right\|_{2,\nu}$$

Subsection 2

Picard's Theorem

Main Theorem

Theorem (Nonlinear Picard-Type Theorem)

Let M be a material law and $\nu > s_b(M)$ and $c > 0$ s.t. $zM(z) - c$ monotone for all $\operatorname{Re} z \geqslant \nu$. Let $A \subseteq L_{2,\nu}(\mathbb{R}, H)^2$ be maximal monotone and autonomous.

(a) *For every $f \in \operatorname{dom}(\partial_{t,\nu})$, there exists unique $u \in L_{2,\nu}(\mathbb{R}, H)$ s.t.*

$$(u, f) \in \partial_{t,\nu} M(\partial_{t,\nu}) + A.$$

(b) *For every $f \in L_{2,\nu}(\mathbb{R}, H)$, there exists unique $u \in L_{2,\nu}(\mathbb{R}, H)$ s.t.*

$$(u, f) \in \overline{\partial_{t,\nu} M(\partial_{t,\nu}) + A}.$$

In both cases, u depends Lipschitz-continuously on f with Lipschitz-constant c^{-1}.

Proof of (a).

Recall: $\|u-v\| \leqslant \frac{1}{c}\|f-g\|$ for $(u,f),(v,g) \in B$ if $B-c$ mon.

$\partial_{t,\nu} M(\partial_{t,\nu}) + A - c$ mon. $\implies$ uniqueness, Lipschitz-cont. dep.

$\to$ *need to show existence*

Have seen: $\partial_{t,\nu} M(\partial_{t,\nu}) - c$ is **maximal** monotone

Strategy: Use perturbation result from earlier using Yosida approximation A_λ, $\lambda > 0$

$\partial_{t,\nu} M(\partial_{t,\nu}) - c,\ A$ max. mon. $\implies \partial_{t,\nu} M(\partial_{t,\nu}) + A_\lambda - c$ max. mon.

$\overset{\text{Minty}}{\implies}$ surjectivity: $\forall f\, \exists u_\lambda : (u_\lambda, f) \in \partial_{t,\nu} M(\partial_{t,\nu}) + A_\lambda$

$$\begin{aligned}\partial_{t,\nu} M(\partial_{t,\nu}),\ A \text{ autonomous} &\implies (\tau_h u_\lambda, \tau_h f) \in \partial_{t,\nu} M(\partial_{t,\nu}) + A_\lambda \\ &\implies \|\tau_h u_\lambda - u_\lambda\| \leqslant \frac{1}{c}\|\tau_h f - f\|\end{aligned}$$

Proof of (a).

Per assumption $f \in \mathrm{dom}(\partial_{t,\nu})$, hence

$$\left\| \frac{\tau_h u_\lambda - u_\lambda}{h} \right\| \leqslant \frac{1}{c} \left\| \frac{\tau_h f - f}{h} \right\| \leqslant \frac{e^{h\nu}-1}{ch\nu} \left\| \partial_{t,\nu} f \right\|$$

$$\begin{aligned} \left\| \frac{\tau_h - \mathrm{id}}{h} M(\partial_{t,\nu}) u_\lambda \right\| &= \left\| M(\partial_{t,\nu}) \frac{\tau_h u_\lambda - u_\lambda}{h} \right\| \\ &\leqslant \left\| M(\partial_{t,\nu}) \right\| \frac{e^{h\nu}-1}{ch\nu} \left\| \partial_{t,\nu} f \right\| \end{aligned}$$

$$\implies \left\| \partial_{t,\nu} M(\partial_{t,\nu}) u_\lambda \right\| \leqslant \left\| M(\partial_{t,\nu}) \right\| c^{-1} \left\| \partial_{t,\nu} f \right\|$$

Then $(u_\lambda, f) \in \partial_{t,\nu} M(\partial_{t,\nu}) + A_\lambda$ implies

$$\limsup_{\lambda \to 0} \left\| A_\lambda(u_\lambda) \right\| \leqslant \limsup_{\lambda \to 0} \underbrace{\left(\left\| f \right\| + \left\| M(\partial_{t,\nu}) \right\| c^{-1} \left\| \partial_{t,\nu} f \right\| \right)}_{\textit{indep. of } \lambda} < \infty$$

$\to$ *We can apply result on perturbation via Yosida approx.!*

End of Proof (a). Proof of (a) $\Longrightarrow$ (b).

By perturbation result, obtain $u \in L_{2,\nu}$ with

$$(u, f) \in c + (\partial_{t,\nu} M(\partial_{t,\nu}) - c) + A = \partial_{t,\nu} M(\partial_{t,\nu}) + A$$

(a) $\Longrightarrow$ (b):

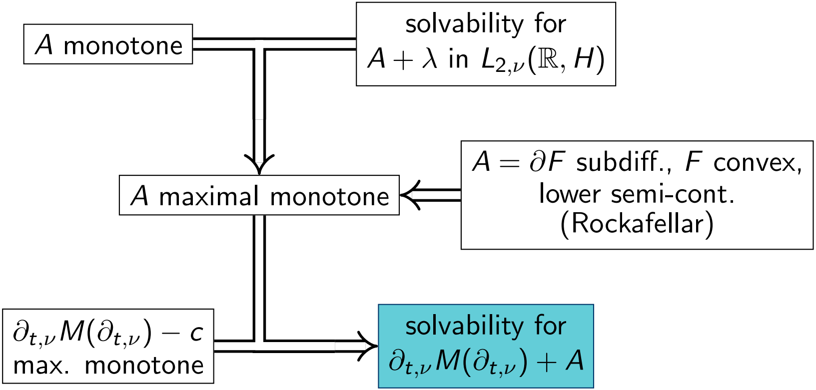
$\partial_{t,\nu} M(\partial_{t,\nu}) + A$ has dense range containing $\text{dom}(\partial_{t,\nu})$ and $\partial_{t,\nu} M(\partial_{t,\nu}) + A - c$ is monotone

$\overset{\text{Cor. of Minty}}{\Longrightarrow}$ $\overline{\partial_{t,\nu} M(\partial_{t,\nu}) + A}$ is surjective $\square$

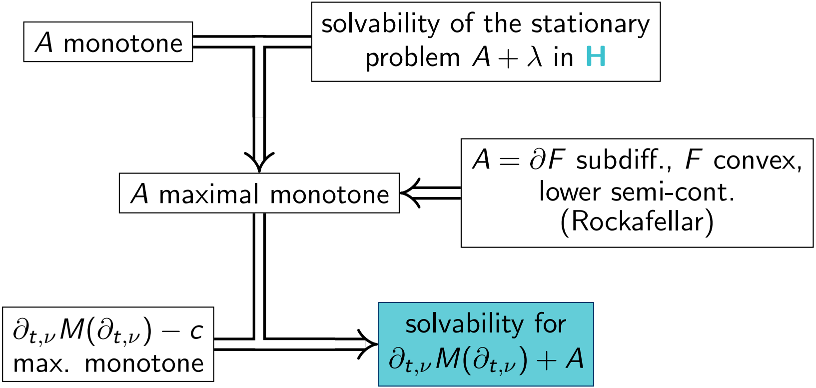
Section 3

Applications

General Scheme



. . . if A is time-independent:



Free Form Architecture

- Minimization of material cost = surface area $S(u)$
- Curvature stress counteracting inner and outer forces (fixation, gravity, etc.)

$\rightarrow$ *model by PDE on mean curvature* $-\frac{1}{2}Q(u)$ *with forces* f

$\rightarrow$ *discussion of PDE gives error estimates and stability of adapted Newton method*

Mean Curvature Operator

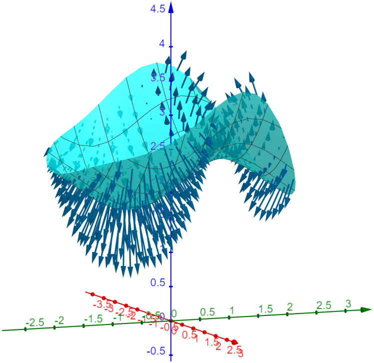


Figure:

$Q(u) \cdot \nu$ on a surface $(x, u(x))$,
$u : \Omega \subseteq \mathbb{R}^2 \to \mathbb{R}$,
$Q(u) = -2H$ for mean curvature
H w.r.t. upwards normal ν

Mean Curvature Operator Q in $L^2(\Omega)$

$$Q(u) = -\operatorname{div}\left(\frac{\nabla u}{\sqrt{1 + |\nabla u|^2}}\right)$$

is monotone by the following result:

Modified Laplacian

Proposition

Let $\Omega \subseteq \mathbb{R}^n$ be a bounded domain with C^1-boundary, $f : \mathbb{R}^n \to \mathbb{R}^n$ monotone and C^1. Then the following operator is monotone in $L^2(\Omega)$: Let Q^f be the closed operator defined by

$$\boxed{Q^f(u) := -\operatorname{div}\left(f(\operatorname{grad} u)\right)}$$

on the core

$$C^2_{b.c.}(Q^f) := \{u \in C^2(\overline{\Omega}) : \begin{array}{r} u = g_D \ \text{ on } \partial\Omega_D \\ f(\nabla u) \cdot \nu = g_N \ \text{ on } \partial\Omega_N \} \end{array}$$

where we decomposed $\partial\Omega = \partial\Omega_D \,\dot\cup\, \partial\Omega_N$ into Borel sets and prescribed functions $g_D \in C^2(\partial\Omega_D)$, $g_N \in C^1(\partial\Omega_N)$.

Modified Laplacian

$H := L^2(\Omega)$. For all $u, v \in C^2_{b.c.}(Q^f)$:

$$\left\langle Q^f(u) - Q^f(v), u - v \right\rangle_H = \langle f(\nabla u) - f(\nabla v), \nabla u - \nabla v \rangle_H + \int_{\partial\Omega} \underbrace{(u-v)(f(\nabla u) - f(\nabla v)) \cdot \nu}_{\equiv 0} \, \mathrm{d}x$$

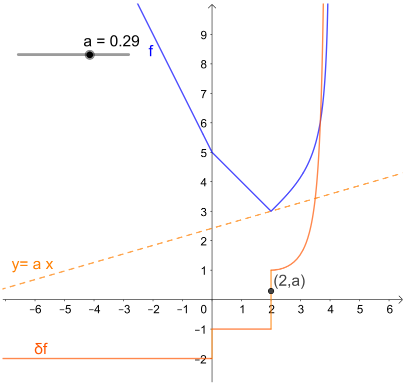
$$\geqslant 0 \qquad (f \text{ monotone})$$

Hence, the closure Q^f is monotone.

Question

Q^f lies in some maximal monotone relation (Zorn), but how to choose it?

- If one is lucky, $\overline{Q^f}$ is surjective.
- Maybe $Q^f \subseteq \partial S$ for some S convex, lower-semicont.



$$\partial S = \{(u,y) : \forall v \in H : S(v) \geqslant \underbrace{S(u) + \langle y\,,\, v-u \rangle}_{\text{hyperplane through } (u, S(u))}\}$$

Mean Curvature Operator (cont'd)

As before, $\Omega \subseteq \mathbb{R}^2$ is bounded with C^1-boundary.

$F(x) := \sqrt{1+|x|^2}$, then $f(x) = \nabla F(x)$ is monotone on $\mathbb{R}^2$.

Recall $Q : \text{dom}(Q) \to L^2(\Omega)$, $Q(u) = -\operatorname{div}\left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}}\right)$

From now on, assume $\partial\Omega_N = \emptyset$ (otherwise have to modify S)

First formal guess: $\boxed{S(u) = \int_\Omega \sqrt{1+|\nabla u|^2}}$ **Area Functional**.

For an arbitrary variation $\xi = v - u$ with $u, v \in C^2_{b.c.}(Q)$:

$$\delta_\xi S(u) = \int_\Omega \frac{\langle \nabla u, \nabla \xi \rangle}{\sqrt{1+|\nabla u|^2}}\,\mathrm{d}x = \int_\Omega Q(u)\xi\,\mathrm{d}x$$

$\implies$ Q is Gâteaux derivative at all u where it exists.

Natural domain of S: $\{u \in W^{1,1}(\Omega) : u|_{\partial\Omega} = g_D\} \subseteq L^2(\Omega)$ by Sobolev embedding. Can we extend to $u \in L^2(\Omega)$ by $S(u) := +\infty$?

- ✓ $Q \subseteq \partial S$
- ✓ S is convex
- × **Problem: lower semi-continuity**

Trick: epigraph-completion. Define for all $u \in L^2(\Omega)$:

$$\tilde{S}(u) := \inf\{r \in \mathbb{R} : (u, r) \in \overline{\mathrm{epi}(S)}\}$$

- ✓ lower semi-continuous per definition
- ✓ $\tilde{S}$ is convex because $\overline{\mathrm{epi}(S)} \subseteq H \times \mathbb{R}$ is convex
- ✓ $Q \subseteq \partial\tilde{S}$
- × Boundary conditions not at all clear.

Application of The Main Theorem

The Main Theorem gives the existence of $L_{2,\nu}(L^2(\Omega))$ generalized mean curvature flow u

$$(\partial_{t,\nu} + \partial\tilde{S}) \ni (u, f)$$

for right hand side $f \in \operatorname{dom}(\partial_{t,\nu}) \subseteq L_{2,\nu}(L^2(\Omega))$.

In particular, $u \in \operatorname{dom}(\partial\tilde{S}) \cap \operatorname{dom}(\partial_{t,\nu})$, **but what is** $\mathbf{dom}(\partial\tilde{S})$**?**
Idea: $\tilde{S}(v) \geqslant \tilde{S}(u) + \langle y\,, v - u\rangle$ not satisfiable for all $v \in L^2(\Omega)$ if $\tilde{S}(u) = +\infty$

Summary

- (Maximal) monotone relations, examples in $\mathbb{R} \times \mathbb{R}$, skew-s.a., coercivity cond., Rockafellar's Theorem
- Minty's Theorem, Corollary for strictly monotone with dense range
- Perturbation by Yosida approximation
- Nonlinear Picard-type Theorem
- Application to mean curvature flow with outer forces

Outlook

- Eventual Independence of $\mu \geqslant \nu_0$
- Continuous dependence on the material law
- Solution theory on the half line $\mathbb{R}_{\geqslant 0}$

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THANK YOU FOR YOUR ATTENTION AND HAVE A NICE ISEM!