23rd Internet Seminar
“Evolutionary Equations”
Final Workshop[1]

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June 22 to June 26, 2020

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Programme

The Workshop takes place in Zoom:

`https://us02web.zoom.us/j/85080697102?pwd=ekh2ckRSY3pnUDRzVGR2b1Z5Z2Q1QT09`

Meeting-ID: 850 8069 7102
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Programme

Isem 23 Phase 3 Workshop June 22, 2020 to June 26, 2020 via Zoom

times: GMT+2:00

from	to	Monday	Tuesday	Wednesday	Thursday	Friday
09:00	09:15	Opening				
09:15	09:30	Plenary Talk Picard	Project K	Plenary Talk Pauly	Plenary Talk Weber	Project F
09:30	09:45					
09:45	10:00					
10:00	10:15					
10:15	10:30					
10:30	10:45					
10:45	11:00	Project L		Project I	Project E	
11:00	11:15		Project H			Project C
11:15	11:30					
11:30	11:45					
11:45	12:00					
12:00	12:15					
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12:45	13:00					
13:00	13:15					
13:15	13:30					
13:30	13:45					
13:45	14:00					
14:00	14:15	Project A	Project N	Free Afternoon	Project B	Project D
14:15	14:30					
14:30	14:45					
14:45	15:00					
15:00	15:15					
15:15	15:30					Closing
15:30	15:45					
15:45	16:00	Project M	Project P		Coordinators Meeting	
16:00	16:15					
16:15	16:30					
16:30	16:45					
16:45	17:00					
17:00	17:15					
17:15	17:30					
17:30	17:45					
17:45	18:00					
18:00	18:15			Virtual Conference Dinner		
18:15	18:30					
18:30	18:45					
18:45	19:00					

Plenary Talks

On Abstract Friedrichs Systems (The Skew-Selfadjoint Case)

Rainer Picard

We shall discuss a particular subclass of evolutionary equations, which are closely linked to the classical concept of Friedrichs systems. We shall focus here on the skew-selfadjoint case, i.e. equations of the form

$$\overline{(\partial_0 M_0 + M_1 + A)}U = F,$$

where A is skew-selfadjoint. Skew-selfadjointness is a rather subtle operator property and we shall inspect several ways of interest in applications to obtain such operators.

Static Solution Theory

Dirk Pauly

The main body of [4] dealt with the notion of evolutionary equations which provided a rich solution theory for time-dependent problems. It is the aim of this talk to complement this time-dependent solution theory with an extension to time-independent (i.e., static) problems, such as the following prototypical electro static system

$$\mathrm{curl}_0 E = F, \qquad \mathrm{div} E = g.$$

Employing techniques similar to those used in [4, Section 11.3] we will develop a comprehensive solution theory for static problems of the above type. We will introduce the notion of Hilbert complexes

$$H_0 \xrightarrow{A_0} H_1 \xrightarrow{A_1} H_2,$$

of densely defined and closed linear operators

$$A_0 : \mathrm{dom}(A_0) \subseteq H_0 \to H_1, \qquad A_1 : \mathrm{dom}(A_1) \subseteq H_1 \to H_2,$$

where the so-called complex property

$$\mathrm{ran}(A_0) \subseteq \ker(A_1)$$

is satisfied. The above electro static system is then generalised to

$$A_1 x = f, \qquad A_0^* x = g.$$

The aim is to provide criteria on the complex such that existence and uniqueness of x can be guaranteed. For this we follow the rationale of [1, 2, 3]

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Dynamical systems and crossed products of $C*$-algebras

Moritz Weber

The theory of $C*$-algebras provides good tools for studying dynamical systems, i.e. actions of compact groups on compact spaces. Typically these $C*$-algebras are noncommutative and we are in the realm of "noncommutative topology". I will give a brief introduction to the theory of $C*$-algebras and the general philosophy of "noncommutative or quantum mathematics" building on Gelfand duality. We then pass to the crossed product construction for investigating dynamical systems. I will survey some of these aspects and mention some examples such as the irrational rotation algebra (also called the non-commutative torus). In this talk, we do not assume any knowledge on $C*$-algebras. This talk can be seen as a teaser for ISem24, see also the webpage:

`https://www.math.uni-sb.de/ag/speicher/ISem24.html`

Projects

Project A

A Hilbert space approach to fractional differential equations

Project Coordinator(s): Sebastian Bechtel, Jan Meichsner

Participants: Ismail Huseynov, Adama Ndoye, Lars Niedorf

The project will be centered around [2]. There, the theory of evolutionary equations is applied to fractional ODEs, that is to say, ordinary differential equations in which a fractional time derivative $\partial_{t,\nu}^{\alpha}$ appears on the left-hand side instead of the usual (full) time derivative. We consider two models that originate from different choices of initial data. The material law from [1, Lecture 5, Example 5.3.1 (e) and 5.3.3 (e)] as well as the extrapolation space of $\partial_{t,\nu}^{\alpha}$ will play important roles.

Depending on the interests of the participants, the project may be extended by additional topics such as general fractional powers of operators or interpolation spaces.

Bibliography

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Project B
Semigroups and Evolutionary Equations

Project Coordinator(s): Christian Budde

Participants: Markus Borkowski, Mehmet Erbay, Marianna Porfido

In this project we want to combine the main objects of this years Internetseminar, namely the *Evolutionary Equations*, with the theory of *strongly continuous one-parameter operator semigroups*, or *C_0-semigroups* for short, which arise in the context of evolution equations. Semigroups and evolution equations are intensively treated for example by Engel and Nagel [2], Goldstein [3] or Pazy [6], just to mention a few. By definition a family $(T(t))_{t\geq 0}$ of bounded linear operators on a Hilbert space $\mathcal{H}$ is called a C_0-semigroup if $T(t+s) = T(t)T(s)$ for all $t, s \geq 0$, $T(0) = \mathrm{I}$ and $\|T(t)x - x\| \to 0$ for all $x \in \mathcal{H}$. To each semigroup one can associate an unbounded operator $(A, \mathrm{D}(A))$, called the *generator*. The reverse direction is part of the so-called Hille–Yosida theorem.

We will consider a well-posed evolutionary problem which is associated with (M, A) and construct a corresponding C_0-semigroup on a certain function space. To do so we need to define the space of admissible history functions and initial states. Moreover, we have to take extrapolation spaces into account, which by construction are completions of the underlying space, cf. [4], [5] or [1]. Time permitting, we will treat differential-algebraic equations in infinite dimensions and concrete hyperbolic delay equations. This project is based on work by S. Trostorff, cf. [7] and [8].

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Project C

Da Prato and Grisvard's approach to define sums of sectorial operators, and maximal regularity

Project Coordinator(s): Ralph Chill, Sebastian Mildner

Participants: Dorothea von Criegern, Al Depope, Bilel Elgabeur

In this project we review some ideas from a seminal paper of Da Prato and Grisvard from 1975. Motivated by applications to linear Cauchy problems, and like in this year's internet seminar, Da Prato and Grisvard studied sums of two unbounded (commuting) sectorial operators on a Banach space, and the related problems of wellposedness of Cauchy problems and the so-called maximal regularity.

The project has three parts:

I. Given two commuting unbounded, invertible and sectorial operators A and B on a Banach space satisfying an angle condition, we define the sum $A + B$ (or actually, the closure of the sum) by writing down the inverse S of the sum with the help of a general form of Cauchy's integral formula:

$$S := \frac{1}{2\pi i} \int_\gamma (z + B)^{-1} (z - A)^{-1} \, dz,$$

where γ is an appropriately chosen curve. In the first part of the project, we motivate this formula and prove that this bounded operator S is the good candidate for the inverse of the closure of $A + B$.

II. In several applications, one would like to show that S is the inverse of $A + B$ itself, and not only of the closure of $A + B$. In other words, one would like to show that the sum $A + B$ on the natural domain, that is, the intersection of the domains of A and B, is already a closed, linear operator. Sometimes one speaks then of maximal regularity. In the second part of the project, we show that the sum $A + B$ is always closed in certain interpolation spaces between X and the domain of A (or of B). In other words, maximal regularity always holds in interpolation spaces (but not always in the space X itself).

III. We apply the results from parts I and II to study wellposedness and maximal regularity of linear Cauchy problems of the form $\dot{u} + Au = f$, $u(0) = 0$.

Bibliography

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Project D
Quantitative Homogenisation Theory

Project Coordinator(s): Shane Cooper

Participants: Imane Essadeq, Salem Nafiri

In ISem23 Lecture 14, amongst other things, homogenisation for the wave equation was studied; in particular the solution $u_n \in L_{2,\nu}(\mathbb{R}; L^2(Y))$ to

$$\partial_{t,\nu}^2 u_n - \mathrm{div}_{\#} a(nm) \mathrm{grad}_{\#} u_n = f \in L_{2,\nu}(\mathbb{R}; L^2(Y)),$$

was shown to converge strongly in L^2 to the solution u_0 of the homogenised equation:

$$\partial_{t,\nu}^2 u - \mathrm{div}_{\#} a_{\mathrm{hom}} \mathrm{grad}_{\#} u = f \in L_{2,\nu}(\mathbb{R}; L^2(Y)).$$

Much more can be said; for example the L^2-norm of the difference $r_n := u_n - u$ is known to be bounded from above by $C\frac{1}{n}\|f\|_{L_{2,\nu}(\mathbb{R};L^2(Y))}$, where the constant C depends only on a and Y. Such quantitative statements are called error estimates in homogenisation. Additionally, so-called corrector estimates exist: here one seeks to determine corrections to u that allow for improvements to the error estimate. For example one can aim to control the difference in a stronger norm (for example the $L_{2,\nu}(\mathbb{R}; H^1_{\#}(Y))$-norm). Or, seek corrections to improve the above rate of convergence in L^2-norm from $\frac{1}{n}$ to $\left(\frac{1}{n}\right)^2$. Such types of quantitative homogenisation results began to appear in the literature as early as 2003/4, in [2], for L^2 type estimates, and 2005/6 in [3], for H^1 corrector-type estimates.

Error estimates in homogenisation have since been proved, by various means, in various contexts. However, even now it is still an important research activity to revisit these results and develop further our understanding on the underlying fundamental concepts of their proof.

In [4] a novel approach to proving error estimates by combining quantitative homogenisation theory with evolutionary equations appeared. In this project we will study and learn this approach in the context of the wave equation. If time permits, we shall see how this approach allows one to determine corrector-type estimates such as those mentioned above.

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Project E

Theorem of Widder–Arendt

Project Coordinator(s): Alexander Dobrick, Federica Gregorio, Abdelaziz Rhandi

Participants: Fabian Gabel, Markus Hartlapp, Lorenzo Lamberti

In 1934 Widder [3] proved the following characterization of Laplace transforms of real-valued bounded functions:

Theorem (Widder)**.** *Let r be a real-valued C^∞-function on $(0,\infty)$. Then there exists $f \in L^\infty(0,\infty)$ such that*

$$r(\lambda) = \int_0^\infty \mathrm{e}^{-\lambda t} f(t)\,\mathrm{d}t, \quad \lambda > 0,$$

if and only if

$$\sup_{\lambda>0,\, n\in\mathbb{N}} \left| \frac{\lambda^{n+1} r^{(n)}(\lambda)}{n!} \right| < \infty.$$

The aim of this project is to extend Widder's theorem to Banach space valued functions. This extension is due to W. Arendt [1].

More precisely, it can be proved that such an extension result holds if and only if the Banach space has the so called Radon–Nikodým property. Since not all Banach spaces have the the Radon–Nikodým property, it is natural to look for a more general version of Widder's theorem. As it turns out, one can show that an "integrated version" of Widder's theorem holds in arbitrary Banach spaces. Also applications to inhomogeneous Cauchy problems will be treated in the project.

More details can be found in [2].

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Project F

Multiplication Operators on Spaces of Vector-Valued Functions

Project Coordinator(s): Retha Heymann

Participants: Patrizio Bifulco, Samir Boujijane, Maximilian Weinberg, Alexander Wittenstein

Multiplication operators on function spaces are of interest in various settings. The Spectral Theorem states that every bounded self-adjoint operator on a Hilbert space is unitarily equivalent to a multiplication operator on an L^2-space. Matrix multipliers are studied in Control Theory. Multiplication operators on L^p-Spaces have elegant properties related to the inducing function and provide valuable examples and counterexamples in various settings. The generalisation to Bochner Spaces follows naturally.

There have been several examples of and comments on multiplication operators in the lecture notes of this internet seminar (e.g., [4, Lecture 2]). In [4, Section 5.2] we saw that "multiplication by V"-operators are defined on $L^2(\mathbb{R}, H)$-spaces with H a Hilbert space. This led to one of the main concepts needed to understand evolutionary equations: the notion of material laws and material law operators.

In order to understand operators being defined by material laws similar to the way it has been done in the internet seminar, in this project, we will focus on multiplication operators on $L^2\big((\Omega, \Sigma, \mu), X\big)$ where (Ω, Σ, μ) is a complete measure space and X is a Banach space. In a similar way to the rationale being carried out in the concluding section of [4, Lecture 2], where we related spectral properties of the multiplication operator to the values of the function we are multiplying, we will study how some properties of such a multiplication operator are related to the so-called "pointwise" operators. The plan is that we will start by studying matrix multipliers (see [2]) and then move on to the more general setting. We could consider eigenvalues, stability properties, decomposition results (see [3]) and the generalisation to so-called fibre spaces ([1]).

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Project H

Modelling of dynamical systems on networks as port-Hamiltonian systems

Project Coordinator(s): Ivica Nakić

Participants: Yassine Kharou, Johannes Rolf Stojanow

Port-Hamiltonian systems are a special kind of differential-algebraic equations, similar to those introduced in [1, Lecture 10], which are endowed with a special mathematical structure. In the port-Hamiltonian systems theory the physical system is seen as an interconnection of simple subsystems, mutually influencing each other via energy flow, hence this framework is suitable for modelling and analysis of multi-physics systems.

The dynamical systems on networks are dynamical systems consisting of simpler subsystems mutually linked in a network-like fashion where the dynamics arises as a compound effect of the interaction between sub-systems.

The first goal of the project is to introduce the relevant terminology of the port-Hamiltonian systems theory including the so-called Dirac structure. The second goal is to examine how this framework can be used to model and analyse dynamical systems on networks. The main source is [2].

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Project I

Picard-Weber-Weck Selection Theorem

Project Coordinator(s): Frank Osterbrink, Dirk Pauly, Michael Schomburg

Participants: Titus Pinta, Nathanael Skrepek

In [5, Chapter 14] we have seen that compact embedding results are important for obtaining homogenisation results for evolutionary equations. In [5] we have focussed on the wave and heat equation or variants thereof. If one wants to obtain homogenisation results also for Maxwell's equations, more sophisticated compactness results than Rellich-Kondrachov's selection theorem [5, Theorem 14.2.5] are needed, see [6, Example 7.12].

This project is about proving the following remarkable theorem [4, 7, 8].

Theorem 1 (Picard-Weber-Weck selection theorem). *Let $\Omega \subset \mathbb{R}^3$ be a bounded weak Lipschitz domain. Then*

$$H_0(\mathrm{curl}, \Omega) \cap H(\mathrm{div}, \Omega) \hookrightarrow L_2(\Omega)^3$$

compactly.

This result is indeed remarkable since it avoids undue regularity constraints on the domain Ω. Thus, quite naturally, we will discuss the notion of a weak Lipschitz domain. Moreover, Helmholtz decompositions and regular potentials are the crucial tools for the proof, which we take from [1, 2, 3].

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Project K

Evolutionary equations and DAEs

Project Coordinator(s): Felix Schwenninger

Participants: JaeYong Choi, Michael Doherty, Annika Meyer

In Lecture 10 we have already encountered differential algebraic equations (DAEs) in the context of evolutionary equations. In this project we continue to study this class and aim for relating existing results in the context of partial-differential-algebraic equations. The topics included are the relation between DAEs and strongly continuous semigroups [5] as well as examples of DAEs in infinite dimensions such as appearing in electrical circuits [2, 3] or heat-wave systems [1, 4, 7].

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Project L

Nonlinear evolutionary equations

Project Coordinator(s): Hendrik Vogt

Participants: Sahiba Arora, Fernanda Florido-Calvo, Gabriel McCracken

In the ISem lectures we studied evolutionary equations of the form

$$\big(\partial_{t,\nu} M(\partial_{t,\nu}) + A\big)U = F,$$

with a material law M and a *skew-selfadjoint* operator A in a Hilbert space H. The aim of this project is to investigate the solution theory for the more general case of *maximal monotone* relations in H; then the above equation becomes an *inclusion*,

$$\big(\partial_{t,\nu} M(\partial_{t,\nu}) + A\big)U \ni F,$$

which can also be written as $\partial_{t,\nu} M(\partial_{t,\nu}) + A \ni (U, F)$.

A linear relation A in H is called *monotone* if

$$\operatorname{Re}\langle u - v, x - y\rangle \geq 0 \qquad \big((u, x), (v, y) \in A\big),$$

and it is *maximal monotone* if it has no proper extension that is monotone as well. In the case of linear operators, maximal monotonicity is the same as m-accretivity (and skew-selfadjoint operators are m-accretive).

Starting point of the project are the papers [1, 2]. Both papers deal with the nonlinear evolutionary inclusion described above, and they discuss applications to nonlinear problems of mathematical physics. The first paper treats material laws of the special form $M = M_0 + z^{-1}M_1$, the second one contains the generalisation to arbitrary material laws.

Important introductory topics shall be Minty's theorem, which characterises maximal monotone relations, and perturbation theory for monotone relations. The core of the project is the solution theory for the above evolutionary inclusion, plus applications. The precise selection of topics will be decided among the participants of the project.

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Project M

Exponential Stability for Hyperbolic Problems

Project Coordinator(s): Jürgen Voigt

Participants: René Hosfeld

The objective of the project is studying equations of the form

$$(\partial_t^2 M(\partial_t) + C^*C)u = f, \tag{1}$$

where the operator $C\colon \operatorname{dom}(C) \subseteq H_0 \to H_1$, with Hilbert spaces H_0, H_1 has suitable properties and the material law M has a special form; see [3, Section 2.2] for more details. The project is a continuation of Lecture 11 from [1]; in fact the procedure applied in the treatment is to transform (1) into an equation of the type

$$(\partial_t M_d(\partial_t) + A)U = F, \tag{2}$$

treated in the ISem Lecture Notes, [1, Section 6.2], and then exponential stability for (1) is defined as exponential stability for (2) in the sense of [1, Lecture 11].

The main source for the project will be [3], and there specifically:
(a) Sec. 2.2 Exponential stability for a class of second order evolutionary problems (However, in order to get the main results, Props. 2.2.3 and 2.2.5, one also has to study first Props. 2.1.5 and 2.1.6 from Sec. 2.1. This is because of the transition from the equation (1) to (2), mentioned above.)
(b) Sec. 2.3.2 Exponential stability for equations of hyperbolic type (Here I suggest to look first at the second example, "Abstract damped wave equation", and then maybe at the first, "Dual phase lag heat conduction".)

For getting additional information, it might also be reasonable to have a look at [2], in particular [2, Sec. 3.1]. The references [2] and [3] can be found at
`http://www.math.tu-dresden.de/~voigt/isem23/`

Comments on notation. The operators $\partial_{t,\nu}$ of [1] are denoted as $\partial_{0,\rho}$ in [2] and [3]. "Material law" in [3] is slightly more general than in [1], and is called "linear material law", where "linear" refers to the linearity of the operators $M(z)$. In [2] there is even a more remarkable difference to the definition we are used to from [1]: Material laws are defined – loosely speaking – on open sets $\Omega \subseteq \mathbb{C}$, for which $\{z^{-1};\, z \in \Omega\}$ contains a right half-plane; see [2, p. 1011] for details. This has the effect that the operators $\partial_{t,\nu} M(\partial_{t,\nu})$ from [1] occur as $\partial_0 M(\partial_0^{-1})$ in [2].

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Project N

Maximal Monotone Operators and Port-Hamiltonian PDE's

Project Coordinator(s): Sven-Ake Wegner

Participants: Dennis Gallaun, Hannes Gernandt, Mohamed Khazou, Lukas Vorberg

During Phase 1 we studied evolutionary equations of the form

$$(\partial_t M(\partial_t) + A)U = F$$

where A is a skew-selfadjoint operator, given by a block matrix of the form

$$\begin{pmatrix} 0 & D \\ G & 0 \end{pmatrix}$$

with D being the divergence and G being the gradient, restricted to a suitable domain. In this project we get to read the paper [1] by Trostorff in which the above is extended to the case that A is a maximal monotone relation which allows to cover more general D's and G's. As an application we will see how the port-Hamiltonian partial differential equation (see the book [2] by Jacob and Zwart)

$$\frac{\partial x}{\partial t}(\xi, t) = P_1 \frac{\partial}{\partial \xi}(\mathcal{H}(\xi)x(\xi, t)) + P_0\mathcal{H}(\xi)x(\xi, t) \text{ for } t \geq 0 \text{ and } \xi \in [0, 1]$$

can be rewritten into the aforementioned block form and thus classical results on well-posedness of port-Hamiltonian systems can be recovered as a special case of Trostorff's general theory.

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Project P

Stochastic Evolutionary Equations

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The project at hand is devoted to an extension of the (deterministic) solution theory for evolutionary equations ([3, Lecture 6]) to stochastic partial differential equations. More precisely, we will discuss equations of the form

$$(\partial_{t,\nu} M(\partial_{t,\nu}) + A)U = F(U), \tag{3}$$

where in this case $F(U)$ is an implementation of the stochastic integral. The solution theory for equations of the form (3) for uniformly Lipschitz continuous right-hand sides $U \mapsto F(U)$ is rather easy to obtain with the help of a contraction mapping argument. Thus the focus of the project will be to show that the Ito-integral with respect to an infinite-dimensional Brownian motion can be realised as uniformly Lipschitz continuous mappings. The core of the material is covered in [2]; a generalisation can be found in [1].

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