STATIC SOLUTION THEORY

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The main body of [4] dealt with the notion of evolutionary equations which provided a rich solution theory for time-dependent problems. It is the aim of this project to complement this time-dependent solution theory with an extension to time-independent (i.e., static) problems, such as the following prototypical electro static system

$$\operatorname{curl}_0 E = F, \qquad \operatorname{div} E = g.$$

Employing techniques similar to those used in [4, Section 11.3] we will develop a comprehensive solution theory for static problems of the above type. We will introduce the notion of Hilbert complexes

$$H_0 \xrightarrow{A_0} H_1 \xrightarrow{A_1} H_2,$$

of densely defined and closed linear operators

$$A_0 : \operatorname{dom}(A_0) \subseteq H_0 \to H_1, \qquad A_1 : \operatorname{dom}(A_1) \subseteq H_1 \to H_2,$$

where the so-called complex property

$$\operatorname{ran}(A_0) \subseteq \ker(A_1)$$

is satisfied. The above electro static system is then generalised to

$$A_1 x = f, \qquad A_0^* x = g.$$

The aim is to provide criteria on the complex such that existence and uniqueness of x can be guaranteed. For this we follow the rationale of [1, 2, 3]

This project is suited for 3 to 4 students.

References

[1] D. Pauly. *Introduction to Maxwell's Equations.* `https://www.uni-due.de/maxwell/downloads/pauly-intromax.pdf`, 2016.

[2] D. Pauly. *Maxwellsche Gleichungen (ToolBox und mehr).* `https://www.uni-due.de/maxwell/downloads/pauly-toolboxmax.pdf`, 2019.

[3] D. Pauly. Solution theory, variational formulations, and functional a posteriori error estimates for general first order systems with applications to electro-magneto-statics and more. *Numer. Funct. Anal. Optim.*, 41(1):16-112, 2020, https://www.uni-due.de/maxwell/publications/journals/op/nfao01.pdf.

[4] C. Seifert, S. Trostorff, and M. Waurick. *Evolutionary Equations.* 23rd Internetseminar, `https://www.mat.tuhh.de/veranstaltungen/isem23/_media/main_lectures.pdf`.