NON-AUTONOMOUS PROBLEMS

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As we have seen in this years internet seminar, [3], many problems arising in mathematical physics can be written as

$$(\partial_t M_0 + M_1 + A)U = F,$$

and hence, have the shape of an evolutionary equation with a material law given by $M(z) := M_0 + z^{-1}M_1$. Since all operators involved commute with translation with repsect to time, these equations only cover autonomous problems, which in particular, is reflected in the constraint that the material parameters are independent of time. It is the aim of this project to generalise the class of evolutionary equations to cover also certain non-autonomous problems. One way for doing so is to replace the operators M_0, M_1 by operator-valued multiplication operators, so that the corresponding problem then takes the form

$$(\partial_t M_0(\cdot) + M_1(\cdot) + A)U = F.$$

Such problems were considered in [1]. However, the so introduced non-autonomous problems do not cover autonomous evolutionary equations with a general material law. One way out is to consider equations of the form

$$(\partial_t \mathcal{M}_0 + \mathcal{M}_1 + A)U = F,$$

where $\mathcal{M}_0, \mathcal{M}_1$ are suitable operators acting in space-time. This class was addressed in [2] and covers both, non-autonomous problems of the form mentioned above as well as general autonomous problems.

In this project questions of the solvability of these problem are addressed on the lines of Picard's theorem from the lecture, and applications to the Kelvin-Voigt-type model in visco-elasticity, cf. [1], are elaborated.

The methods used are closely related to the ones known from the lecture phase.

This project is suited for 3 to 4 students.

References

[1] Rainer Picard, Sascha Trostorff, Marcus Waurick, and Maria Wehowski. On non-autonomous evolutionary problems. J. Evol. Equ., 13(4):751–776, 2013.

[2] M. Waurick. On non-autonomous integro-differential-algebraic evolutionary problems. Math. Methods Appl. Sci., 38(4):665–676, 2015.

[3] ISem 23 Lecture Notes, 2020.