MULTIPLICATION OPERATORS ON SPACES OF VECTOR-VALUED FUNCTIONS

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Multiplication operators on function spaces are of interest in various settings. The Spectral Theorem states that every bounded self-adjoint operator on a Hilbert space is unitarily equivalent to a multiplication operator on an L^2-space. Matrix multipliers are studied in Control Theory. Multiplication operators on L^p-Spaces have elegant properties related to the inducing function and provide valuable examples and counterexamples in various settings. The generalisation to Bochner Spaces follows naturally.

There have been several examples of and comments on multiplication operators in the lecture notes of this internet seminar (e.g., [4, Lecture 2]). In [4, Section 5.2] we saw that "multiplication by V"-operators are defined on $L^2(\mathbb{R}, H)$-spaces with H a Hilbert space. This led to one of the main concepts needed to understand evolutionary equations: the notion of material laws and material law operators.

In order to understand operators being defined by material laws similar to the way it has been done in the internet seminar, in this project, we will focus on multiplication operators on $L^2\big((\Omega, \Sigma, \mu), X\big)$ where (Ω, Σ, μ) is a complete measure space and X is a Banach space. In a similar way to the rationale being carried out in the concluding section of [4, Lecture 2], where we related spectral properties of the multiplication operator to the values of the function we are multiplying, we will study how some properties of such a multiplication operator are related to the so-called "pointwise" operators. The plan is that we will start by studying matrix multipliers (see [2]) and then move on to the more general setting. We could consider eigenvalues, stability properties, decomposition results (see [3]) and the generalisation to so-called fibre spaces ([1]).

This project is suited for 3 to 4 students.

References

[1] Budde, C., and Heymann, R. Extrapolation of operator-valued multiplication operators. *arXiv* (2019).

[2] Engel, K.-J. *Operator Matrices and Systems of Evolution Equations*. unpublished, 1997.

[3] Heymann, R. *Multiplication operators on Bochner spaces and Banach fibre spaces*. PhD thesis, Eberhard-Karls-Universität Tübingen, 01 2015.

[4] Seifert, C., Trostorff, S., and Waurick, M. *23rd Internet Seminar "Evolutionary Equations"*. Lecture Notes, 2019-2020.