SEMIGROUPS AND EVOLUTIONARY EQUATIONS

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In this project we want to combine the main objects of this years Internetseminar, namely the *Evolutionary Equations*, with the theory of *strongly continuous one-parameter operator semigroups*, or C_0*-semigroups* for short, which arise in the context of evolution equations. Semigroups and evolution equations are intensively treated for example by Engel and Nagel [2], Goldstein [3] or Pazy [6], just to mention a few. By definition a family $(T(t))_{t\geq 0}$ of bounded linear operators on a Hilbert space $\mathcal{H}$ is called a C_0-semigroup if $T(t+s) = T(t)T(s)$ for all $t, s \geq 0$, $T(0) = \mathrm{I}$ and $\|T(t)x - x\| \to 0$ for all $x \in \mathcal{H}$. To each semigroup one can associate an unbounded operator $(A, \mathrm{D}(A))$, called the *generator*. The reverse direction is part of the so-called Hille–Yosida theorem.

We will consider a well-posed evolutionary problem which is associated with (M, A) and construct a corresponding C_0-semigroup on a certain function space. To do so we need to define the space of admissible history functions and initial states. Moreover, we have to take extrapolation spaces into account, which by construction are completions of the underlying space, cf. [4], [5] or [1]. Time permitting, we will treat differential-algebraic equations in infinite dimensions and concrete hyperbolic delay equations. This project is based on work by S. Trostorff, cf. [7] and [8].

This project is suited for 3 to 4 students.

References

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