Outline of the Course (i.e. Phase 1) of the 23rd Internet Seminar

After a short first introductory lecture, we shall discuss the concept of (unbounded) operators predominantly defined in Hilbert spaces in Lecture 2. We define the adjoint of an operator and discuss several properties of adjoints. In Lecture 3, we then provide a brief theory of vector-valued integration and define the most important operator in the whole course: the time-derivative operator acting on functions defined on the reals with values in a Hilbert space. Lecture 4 is concerned with an ad-hoc application of the time-derivative operator, namely a Hilbert space approach to ordinary differential equations. With the introduced time-derivative, we will provide a proof of the classical Picard–Lindelöf theorem and shall explore applications to ordinary differential equations with delay or memory terms. Also we will introduce another core principle: causality. After the small detour to ordinary differential equations, in Lecture 5 we will focus on what-is-called a material law and we will provide an explicit spectral representation as multiplication operator (with the help of the so-called Fourier–Laplace transformation) for the time-derivative. This spectral representation and the notion of material laws will help us to properly define 'evolutionary equations' in Lecture 6. In this lecture, we shall also provide a proof of the solution theory for evolutionary equations (i.e., existence and uniqueness of solutions as well as continuous dependence on the data). This lecture will also be accompanied by three examples that well fit into the theory: the heat equation, the wave equation and Maxwell's equations. The versatility of the so far developed theory will be stressed in Lecture 7, where we discuss equations describing port-elastic deformations, fractional elasticity, heat conduction with delay or the so-called dual phase lag model for heat conduction. These are mostly coupled partial differential equations at best formulated in a space-time setting, which is at the heart of the notion of evolutionary equations. Lecture 8 will be much less applied and focusses on an alternative way of showing causality for the solution operator of evolutionary equations. In fact, causality will be the main reason, why it is possible to provide a perspective to initial value problems in Lecture 9. For this, we need to understand, how initial values can be attained at all. Note that evolutionary equations also cover a class of time-independent partial differential equations, where the corresponding initial value problems might not be a sensible thing to solve for. A more detailed discussion of initial value problems for (ordinary) differential algebraic equations in both finite- as well as infinite-dimensional state spaces will be provided in Lecture 10. In Lecture 11, we will be concerned with a qualitative property of solutions of evolutionary equations: exponential stability. For this, the theory developed in the previous lectures will be enormously helpful. Having discussed initial value problems already, we shall focus on inhomogeneous boundary value problems in Lecture 12. As we want to keep the setting as applicable as possible, we will introduce a replacement for trace spaces that are independent of the regularity of the spatial domain a partial differential equation is considered in. Lecture 13 is concerned with another quantitative aspect of the solution operator of an evolutionary equation: the dependence of the solution operator on its coefficients. As an application, we present classical homogenisation results. The concluding Lecture 14 will provide an outlook to further studies and contains miscellaneous facts about evolutionary equations, which may be addressed in the project phase.
Each lecture will be added approximately 7 exercises of varying difficulty.