

# MAXIMAL MONOTONE OPERATORS AND PORT-HAMILTONIAN PDE'S

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During Phase 1 we studied evolutionary equations of the form

$$(\partial_t M(\partial_t) + A)U = F$$

where  $A$  is a skew-selfadjoint operator, given by a block matrix of the form

$$\begin{pmatrix} 0 & D \\ G & 0 \end{pmatrix}$$

with  $D$  being the divergence and  $G$  being the gradient, restricted to a suitable domain. In this project we get to read the paper [1] by Trostorff in which the above is extended to the case that  $A$  is a maximal monotone relation which allows to cover more general  $D$ 's and  $G$ 's. As an application we will see how the port-Hamiltonian partial differential equation (see the book [2] by Jacob and Zwart)

$$\frac{\partial x}{\partial t}(\xi, t) = P_1 \frac{\partial}{\partial \xi}(\mathcal{H}(\xi)x(\xi, t)) + P_0 \mathcal{H}(\xi)x(\xi, t) \text{ for } t \geq 0 \text{ and } \xi \in [0, 1]$$

can be rewritten into the aforementioned block form and thus classical results on well-posedness of port-Hamiltonian systems can be recovered as a special case of Trostorff's general theory.

This project is suited for 3 to 4 students.

## REFERENCES

- [1] S. Trostorff, *A characterization of boundary conditions yielding maximal monotone operators*, J. Funct. Anal. **267** (2014) 2787–2822.
- [2] B. Jacob and H. Zwart, *Linear port-Hamiltonian systems on infinite-dimensional spaces*, Operator Theory: Advances and Applications **223**, Birkhäuser, Basel, 2012.