EXPONENTIAL STABILITY FOR HYPERBOLIC PROBLEMS

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The objective of the project is studying equations of the form

$$(\partial_t^2 M(\partial_t) + C^*C)u = f, \tag{1}$$

where the operator $C\colon \operatorname{dom}(C) \subseteq H_0 \to H_1$, with Hilbert spaces H_0, H_1 has suitable properties and the material law M has a special form; see [3, Section 2.2] for more details. The project is a continuation of Lecture 11 from [1]; in fact the procedure applied in the treatment is to transform (1) into an equation of the type

$$(\partial_t M_d(\partial_t) + A)U = F, \tag{2}$$

treated in the ISem Lecture Notes, [1, Section 6.2], and then exponential stability for (1) is defined as exponential stability for (2) in the sense of [1, Lecture 11].

The main source for the project will be [3], and there specifically:
(a) Sec. 2.2 Exponential stability for a class of second order evolutionary problems (However, in order to get the main results, Props. 2.2.3 and 2.2.5, one also has to study first Props. 2.1.5 and 2.1.6 from Sec. 2.1. This is because of the transition from the equation (1) to (2), mentioned above.)
(b) Sec. 2.3.2 Exponential stability for equations of hyperbolic type (Here I suggest to look first at the second example, “Abstract damped wave equation”, and then maybe at the first, “Dual phase lag heat conduction”.)

For getting additional information, it might also be reasonable to have a look at [2], in particular [2, Sec. 3.1]. The references [2] and [3] can be found at
`http://www.math.tu-dresden.de/~voigt/isem23/`

Comments on notation. The operators $\partial_{t,\nu}$ of [1] are denoted as $\partial_{0,\rho}$ in [2] and [3]. “Material law” in [3] is slightly more general than in [1], and is called “linear material law”, where “linear” refers to the linearity of the operators $M(z)$. In [2] there is even a more remarkable difference to the definition we are used to from [1]: Material laws are defined – loosely speaking – on open sets $\Omega \subseteq \mathbb{C}$, for which $\{z^{-1};\, z \in \Omega\}$ contains a right half-plane; see [2, p. 1011] for details. This has the effect that the operators $\partial_{t,\nu} M(\partial_{t,\nu})$ from [1] occur as $\partial_0 M(\partial_0^{-1})$ in [2].

This project is suited for 3 to 4 students.

References

[1] ISem 23 Lecture Notes, 2020.
[2] Sascha Trostorff: *Exponential stability for second order evolutionary problem.* J. Math. Anal. Appl. **129**, 1007–1032 (2015).
[3] Sascha Trostorff: *Exponential Stability and Initial Value Problems for Evolutionary Equations.* Habilitationsschrift, Fakultät Mathematik und Naturwissenschaften, Technische Universität Dresden, 2017.