# NONLINEAR EVOLUTIONARY EQUATIONS

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In the ISem lectures we studied evolutionary equations of the form

$$\big(\partial_{t,\nu} M(\partial_{t,\nu}) + A\big)U = F,$$

with a material law $M$ and a *skew-selfadjoint* operator $A$ in a Hilbert space $H$. The aim of this project is to investigate the solution theory for the more general case of *maximal monotone* relations in $H$; then the above equation becomes an *inclusion*,

$$\big(\partial_{t,\nu} M(\partial_{t,\nu}) + A\big)U \ni F,$$

which can also be written as $\partial_{t,\nu} M(\partial_{t,\nu}) + A \ni (U, F)$.
A linear relation $A$ in $H$ is called *monotone* if

$$\operatorname{Re}\langle u - v, x - y\rangle \geq 0 \qquad \big((u, x), (v, y) \in A\big),$$

and it is *maximal monotone* if it has no proper extension that is monotone as well. In the case of linear operators, maximal monotonicity is the same as m-accretivity (and skew-selfadjoint operators are m-accretive).
Starting point of the project are the papers [1, 2]. Both papers deal with the nonlinear evolutionary inclusion described above, and they discuss applications to nonlinear problems of mathematical physics. The first paper treats material laws of the special form $M = M_0 + z^{-1}M_1$, the second one contains the generalisation to arbitrary material laws.
Important introductory topics shall be Minty's theorem, which characterises maximal monotone relations, and perturbation theory for monotone relations. The core of the project is the solution theory for the above evolutionary inclusion, plus applications. The precise selection of topics will be decided among the participants of the project.

This project is suited for 3 to 4 students.

## References

[1] S. Trostorff, An alternative approach to well-posedness of a class of differential inclusions in Hilbert spaces. *Nonlinear Anal.* **75** (2012), no. 15, 5851-5865.
[2] S. Trostorff, Autonomous evolutionary inclusions with applications to problems with nonlinear boundary conditions. *IJPAM* **85** (2013), no. 2, 303–338.