PICARD-WEBER-WECK SELECTION THEOREM

FRANK OSTERBRINK, DIRK PAULY, AND MICHAEL SCHOMBURG

In [5, Chapter 14] we have seen that compact embedding results are important for obtaining homogenisation results for evolutionary equations. In [5] we have focussed on the wave and heat equation or variants thereof. If one wants to obtain homogenisation results also for Maxwell's equations, more sophisticated compactness results than Rellich-Kondrachov's selection theorem [5, Theorem 14.2.5] are needed, see [6, Example 7.12].

This project is about proving the following remarkable theorem [4, 7, 8].

Theorem 1 (Picard-Weber-Weck selection theorem). *Let $\Omega \subset \mathbb{R}^3$ be a bounded weak Lipschitz domain. Then*

$$H_0(\operatorname{curl}, \Omega) \cap H(\operatorname{div}, \Omega) \hookrightarrow L_2(\Omega)^3$$

compactly.

This result is indeed remarkable since it avoids undue regularity constraints on the domain Ω. Thus, quite naturally, we will discuss the notion of a weak Lipschitz domain. Moreover, Helmholtz decompositions and regular potentials are the crucial tools for the proof, which we take from [1, 2, 3].

This project is suited for 3 to 4 students.

References

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