THEOREM OF WIDDER-ARENDT

ALEXANDER DOBRICK, FEDERICA GREGORIO, AND ABDELAZIZ RHANDI

In 1934 Widder [3] proved the following characterization of Laplace transforms of real-valued bounded functions:

Theorem (Widder)**.** *Let r be a real-valued C^∞-function on $(0,\infty)$. Then there exists $f \in L^\infty(0,\infty)$ such that*

$$r(\lambda) = \int_0^\infty \mathrm{e}^{-\lambda t} f(t)\,\mathrm{d}t, \quad \lambda > 0,$$

if and only if

$$\sup_{\lambda>0,\, n\in\mathbb{N}} \left| \frac{\lambda^{n+1} r^{(n)}(\lambda)}{n!} \right| < \infty.$$

The aim of this project is to extend Widder's theorem to Banach space valued functions. This extension is due to W. Arendt [1].
More precisely, it can be proved that such an extension result holds if and only if the Banach space has the so called Radon-Nikodým property. Since not all Banach spaces have the the Radon-Nikodým property, it is natural to look for a more general version of Widder's theorem. As it turns out, one can show that an "integrated version" of Widder's theorem holds in arbitrary Banach spaces. Also applications to inhomogeneous Cauchy problems will be treated in the project.

More details can be found in [2].

This project is suited for 3 students.

References

[1] W. Arendt: Vector-valued Laplace transforms and Cauchy problems. Israel J. Math. **59** (1987), 327-352.
[2] W. Arendt, C.J.K. Batty, M. Hieber, F. Neubrander: Vector-Valued Laplace Transforms and Cauchy Problems. Monographs in Mathematics 96, Birkhäuser, 2001.
[3] D.V. Widder: The inversion of the Laplace integral and the related moment problem. Trans. Amer. Math. Soc. **36** (1934), 107-200.