DA PRATO AND GRISVARD'S APPROACH TO DEFINE SUMS OF SECTORIAL OPERATORS, AND MAXIMAL REGULARITY

RALPH CHILL AND SEBASTIAN MILDNER

In this project we review some ideas from a seminal paper of Da Prato and Grisvard from 1975. Motivated by applications to linear Cauchy problems, and like in this year's internet seminar, Da Prato and Grisvard studied sums of two unbounded (commuting) sectorial operators on a Banach space, and the related problems of wellposedness of Cauchy problems and the so-called maximal regularity.

The project has three parts:

I. Given two commuting unbounded, invertible and sectorial operators A and B on a Banach space satisfying an angle condition, we define the sum $A+B$ (or actually, the closure of the sum) by writing down the inverse S of the sum with the help of a general form of Cauchy's integral formula:

$$S := \frac{1}{2\pi i} \int_\gamma (z+B)^{-1}(z-A)^{-1}\, dz,$$

where γ is an appropriately chosen curve. In the first part of the project, we motivate this formula and prove that this bounded operator S is the good candidate for the inverse of the closure of $A+B$.

II. In several applications, one would like to show that S is the inverse of $A+B$ itself, and not only of the closure of $A+B$. In other words, one would like to show that the sum $A+B$ on the natural domain, that is, the intersection of the domains of A and B, is already a closed, linear operator. Sometimes one speaks then of maximal regularity. In the second part of the project, we show that the sum $A+B$ is always closed in certain interpolation spaces between X and the domain of A (or of B). In other words, maximal regularity always holds in interpolation spaces (but not always in the space X itself).

III. We apply the results from parts I and II to study wellposedness and maximal regularity of linear Cauchy problems of the form $\dot{u} + Au = f$, $u(0) = 0$.

This project is suited for 3 to 4 students.

References

[1] G. Da Prato and P. Grisvard, *Sommes d'opérateurs linéaires et équations différentielles opérationnelles*, J. Math. Pures Appl. **54** (1975), 305–387.