The Ornstein–Uhlenbeck operator and semigroup in Gauss space.

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Abstract

In this project the aim is to study the Ornstein–Uhlenbeck operator and semigroup in Gaussian spaces. The starting point will be the theory of Dirichlet forms developed in the lectures and the main reference is the first chapter of the book of Bogachev [1].

We start by fixing a Gaussian measure γ on $\mathbb{R}^d$

$$d\gamma(x) = \frac{1}{(2\pi)^{\frac{d}{2}}} e^{-\frac{|x|^2}{2}} dx.$$

Then we consider the Lebesgue and Sobolev spaces $L^2(\gamma)$ and $W^{1,2}(\gamma)$, and we define the Dirichlet form in $W^{1,2}(\gamma)$

$$a(u,v) = \int_{\mathbb{R}^d} \nabla u(x) \cdot \nabla v(x) d\gamma(x)$$

and the associated (Ornstein–Uhlenbeck) operator L that on regular functions is given by

$$Lu(x) = \Delta u(x) - x \cdot \nabla u(x).$$

We shall characterize the domain $D(L)$ of L in $L^2(\gamma)$. The operator $(L, D(L))$ generates a strongly continuous semigroup in $L^2(\gamma)$ which has the following analytical representation

$$T_t u(x) = \int_{\mathbb{R}^d} u(e^{-t}x + \sqrt{1-e^{-2t}}y) d\gamma(y), \qquad \forall u \in L^2(\gamma).$$

Further results that can be presented are the following:

1. spectrum and eigenfunctions (Hermite polynomials) of L and Wiener chaos decomposition of $L^2(\gamma)$;
2. smoothing properties of the semigroup;
3. functional inequalities (Poincaré and Log–Sobolev) by the semigroup approach.

One of the aims of this project is to give a perspective of the subject of 19^{th} Isem which will focus on the extension of this theory to the infinite dimensional setting of abstract Wiener spaces.

References

[1] V.I.Bogachev: *Gaussian Measures,* American Mathematical Society, 1998.

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