Trotter's Product Formula for Forms

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The aim of this project is to show the famous Trotter's product formula for a certain class of forms. This formula is known in the semigroup and their generators context. It is well known that this formula is very useful to write the "product" semigroup generated by the sum of two operators (or two forms) in terms of their semigroups. It could be used also to approximate numerically the product semigroup. More precisely, we want to show the following.

Theorem 0.1. *Let* $\mathbf{a}$ *and* $\mathbf{b}$ *be closed sectorial forms. Then*

$$s - \lim_{n \to +\infty} (e^{-\frac{t}{n}\mathbf{a}} e^{-\frac{t}{n}\mathbf{b}})^n = e^{-t(\mathbf{a}+\mathbf{b})}, \tag{1}$$

for all $t > 0$, *uniformly on compact subsets of* $(0, \infty)$.

Before showing this result, we study necessary materials as sectorial and symmetric forms, give their relationship with semigroups. We introduce the notion of resolvent convergence sense, and give its caracterization. We show the theorem first fro symmetric forms. We will start working in the case of densely defined forms. For this, we follow the proof of Reed-Simon [5, 6]. If we have time, we can show this theorem in the general case of non densely defined forms. For this last case, we will work on the reference [4]. For some intermediate results, we will need also [1, 3, 2]. At the end, we apply this Formula to some well known elliptic operators.

References

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