Non-autonomous forms: invariance and maximal regularity in H

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We continue with the theory for non-autonomous equations from Lecture 14, extending it in two directions. On the one hand, we generalise the invariance criteria for convex sets to non-autonomous and inhomogeneous Cauchy problems. In this context we present an application to a semi-linear transport problem

$$\begin{cases} u'(t) - \operatorname{div}\big([a_{kl}(t)]\nabla u\big) = u(t)\big(1-u(t)\big), \\ u(0) \in \mathcal{C}, \end{cases}$$

where $\mathcal{C} := \{v \in L^2(\Omega) : 0 \leq v(0) \leq 1 \text{ a.e. on } \Omega\}$. We prove that there exists a unique solution u with maximal regularity such that $u(t) \in \mathcal{C}$ for all $t \in [0,T]$.

On the other hand, we study maximal regularity in H, i.e., the question of whether for every $f \in L^2(0,T;H)$ the solution $u \in \mathrm{MR}(V,V')$ of

$$\begin{cases} u' + \mathcal{A}u = f, \\ u(0) = 0 \end{cases}$$

is in $H^1(0,T;H)$ (and not merely in $H^1(0,T;V^*)$). While this is not true in general, we shall prove that if $\mathcal{A}$ is associated with a symmetric, continuous, coercive non-autonomous form that has *bounded variation*, one does have maximal regularity in H. Here a form $\mathfrak{a}\colon [0,T]\times V\times V \to \mathbb{K}$ is said to be of bounded variation if there exists an increasing function $g\colon [0,T]\to\mathbb{R}$ such that

$$|\mathfrak{a}(t,v,w) - \mathfrak{a}(s,v,w)| \leq \big(g(t)-g(s)\big)\|v\|_V\|w\|_V$$

for all $s \leq t$ in $[0,T]$ and $v,w \in V$.

Both parts of this project are based on elementary methods and no further prerequisites to the ISEM lectures are required to get started. It is noteworthy to remark that the above results are very recent findings that involves topics of active research.

[1] Dominik Dier. *Non-autonomous maximal regularity for forms of bounded variation*. J. Math. Anal. Appl. **425** (2015), no. 1, 33–54.

[2] Wolfgang Arendt, Dominik Dier, and El Maati Ouhabaz. *Invariance of convex sets for non-autonomous evolution equations governed by forms*. J. Lond. Math. Soc. (2) **89** (2014), no. 3, 903–916.

[3] Dominik Dier. *Non-Autonomous Cauchy Problems Governed by Forms: Maximal regularity and Invariance*. PhD dissertation. Ulm University, 2014.