18th Internet Seminar on Evolution Equations

Form Methods for Evolution Equations, and Applications

Workshop Blaubeuren

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Contents

1 Plenary talks

Plenary talk

Gradient systems associated with j-elliptic functionals

Ralph Chill (TU Dresden)

This talk can be seen as an outlook into a theory which presents much similarity with the theory of sectorial forms, their associated operators and the semigroups they generate: the theory of convex functions on Hilbert spaces, their subgradients and the (nonlinear) semigroups they generate. Much of the talk will be a presentation of classical results. This includes wellposedness results as well as Beurling-Deny type criteria for positivity or L^∞-contractivity of semigroups on L^2 and generation of semigroups on L^p. We present in addition an analogue of j-elliptic forms which allow one to define nonlinear versions of Dirichlet-to-Neumann operators. The talk is based in joint work with Daniel Hauer, James Kennedy and Zakaria Belhachmi.

Plenary talk

Functional Calculus based on the Numerical Range

Michel Crouzeix (Université de Rennes 1)

Holomorphic functions of one operator frequently occur in pure as applied mathematics. For instance, the exponential function is related to the semi-group, or the group theory, i.e. in particular to parabolic equations, linear Schroedinger, ... The cosine function plays a similar role for second order evolution equations. For the theory, as well as for the numerical analysis of these problems, estimates of these holomorphic functions are needed. In the case of a self-adjoint operator, spectral theory provides a very efficient tool, but the situation is much more complicated if we consider non normal operators. For them, spectral sets were introduced and studied by J. von Neumann in 1951, but this theory have not known a deep extension up to a work of Bernard and François

Delyon who were the first to discover the role that the numerical range can play for non self-adjoint operators.

Let us consider two complex Hilbert spaces $V \subset H$ (with dense imbedding) and a sesquilinear continuous form $a : V \times V \to \mathbb{C}$. We associate the operator $A \in \mathcal{L}(D(A), H)$ defined by $(Au, v)_H = a(u, v), \forall v \in V$, with

$$D(A) = \{u \in V\,; \exists\, g \in H, (g, v)_H = a(u, v), \forall v \in V\}.$$

By definition, the numerical range $W(A)$ of the operator A is the set

$$W(A) = \{(Au, u)_H\,;\, u \in D(A), (u, u)_H = 1\}.$$

It is a convex subset of the complex plane (Toeplitz-Hausdorff theorem), its closure contains the spectrum $\sigma(A)$. Interesting properties of the sesquilinear form correspond to localization of the numerical range.

In this talk I will show some estimates obtained from assumptions on the numerical range, and I will applied them to second order differential equations..

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Plenary talk

Intrinsic metrics for Dirichlet forms

Daniel Lenz (FSU Jena)

Starting with Sturm's influential work intrinsic metrics have played a prominent role in the investigation of spectral geometry of strongly local Dirichlet forms. Recently, similar concepts have been developped for general Dirichlet forms. This allows one to investigate in particular spectral geometry of graphs whose vertex degree is not uniformly bounded. We give an introduction into the topic.

On convergence of Dirichlet-to-Neumann operators

Tom ter Elst (The University of Auckland)

Let $\Omega \subset \mathbb{R}^n$ be an open bounded set with C^1-boundary. Let $m \in L_\infty(\Omega, \mathbb{R})$. In Lecture 8 the self-adjoint Dirichlet-to-Neumann operator D_m is introduced whenever $0 \notin \sigma(\Delta_D + m)$. Let $\lambda \in \mathbb{C} \setminus \mathbb{R}$. In this talk we shall prove the continuity of the map

$$m \mapsto (\lambda - D_m)^{-1}$$

from $\{m \in L_\infty(\Omega, \mathbb{R}) : 0 \notin \sigma(\Delta_D + m)\}$ into $\mathcal{L}(L_2(\partial\Omega))$ and show that this map extends continuously to the full space $L_\infty(\Omega, \mathbb{R})$. If $m \in L_\infty(\Omega, \mathbb{R})$ and $0 \in \sigma(\Delta_D + m)$, then $\lim_{m' \to m} (\lambda - D_{m'})^{-1}$ turns out to be the resolvent of a self-adjoint graph, not an operator, which we denote by D_m. We shall show that D_m is associated to a form on $H^1(\Omega)$, but for which (8.3) is violated.

One can do the above also for a symmetric operator A in divergence form instead of the Laplacian $-\Delta$ on Ω (cf. Lecture 11). If $m \in L_\infty(\Omega, \mathbb{R})$ and $0 \notin \sigma(-A_D + m)$, then the form method gives a self-adjoint Dirichlet-to-Neumann operator D_m. Again one has continuity of the map

$$m \mapsto (\lambda - D_m)^{-1}$$

from $\{m \in L_\infty(\Omega, \mathbb{R}) : 0 \notin \sigma(-A_D + m)\}$ into $\mathcal{L}(L_2(\partial\Omega))$. This time a continuous extension to the full space $L_\infty(\Omega, \mathbb{R})$ is surprisingly valid if one has the unique continuation property, which implies that the only $u \in D(-A_D + m)$ with $\partial_\nu u = 0$ is the zero function.

This talk is based on joint work with Wolfgang Arendt, James Kennedy and Manfred Sauter.

2 Projects

2.1 Project A

Continuity of the Bogovskiĭ operator via Calderón-Zygmund theory

Christopher Basil George Thornett (Sydney), Pascal van den Bosch (Delft)

Coordinator: Peer Kunstmann (Karlsruhe)

In this project we want to give a proof of Theorem 13.8 of the Internet Seminar on continuity of the Bogovskiĭ operator $B : L_2^0(\Omega) \to H_0^1(\Omega)$ where $\Omega \subseteq R^n$ is a bounded open set that is star-shaped with respect to every point of an open ball and where $L_2^0(\Omega) = \{f \in L^2(\Omega) : \int_\Omega f = 0\}$. Recall that the Bogovskiĭ operator B, originally defined on C^∞-functions with compact support, has the property $\operatorname{div} Bf = f$ for all $f \in L_2^0(\Omega)$. This tells us in particular that any $f \in L_2^0(\Omega)$ is the divergence of a vector field in $H_0^1(\Omega)$. We refer to Lecture 13 of the Internet Seminar for the application of the continuity result to the theory of the Stokes operator.

Continuity of the Bogovskiĭ operator is proved, following the proof of Lemma III.3.1 in [G], by an application of a result from the Calderón-Zygmund theory of singular integral operators [CZ].

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2.2 Project B

Forms and spectral theory: properties of eigenvalues and eigenvectors via forms and semigroups

Fatem Belgacem (Gabes), Lucas Schöbel-Kröhn (Darmstadt), Abdallah Maichine (Salerno), Marcelo Forets (Grenoble)

Coordinator: James Kennedy (Stuttgart)

According to the spectral theorem, any positive, self-adjoint operator A with compact resolvent on a separable Hilbert space H is a diagonal operator, i.e., it has a sequence of eigenvalues $\lambda_n \geq 0$ and corresponding eigenvectors $e_n \in H$ which form an orthonormal basis of H (see Lecture 6 of the Internet Seminar).

Such an operator is also associated with a form a and λ_n and e_n can be characterised purely in terms of the behaviour of a on subsets of its form domain $D(a) =: V$ via the *min-max* and *max-min principles* for the eigenvalues (see [B,CH,RS] and also Exercise 6.3). These formulae are the starting point for a large part of spectral analysis and geometry, and have been used to prove several central results in mathematical physics.

The most famous of these is probably the *Weyl asymptotics* describing the eigenvalues of the Dirichlet or Neumann Laplacian on $L^2(\Omega)$ for a (bounded, sufficiently regular) domain $\Omega \subset \mathbb{R}^d$:

$$\lambda_n \sim c_d \left(|\Omega|^{-1} n\right)^{\frac{2}{d}} \qquad \text{as } n \to \infty,$$

where λ_n is the nth smallest eigenvalue (repeated according to multiplicities), $|\Omega|$ is the volume of Ω and c_d is a dimensional constant (see [RS] or [CH, Ch. VI]).

There is another proof using ideas from semigroup theory. If an operator A (as above) acts on $H = L^2(\Omega)$ and we define

$$k_t(x,y) := \sum_{n=1}^{\infty} e^{-\lambda_n t} e_n(x)\overline{e_n(y)}, \qquad x, y \in \Omega$$

for all $t > 0$ (k_t is called a heat kernel), then according to *Mercer's Theorem*

$$e^{-tA} f(x) = \int_\Omega k_t(x,y) f(y)\,\mathrm{d}y$$

for all $f \in L^2(\Omega)$. It is not hard to prove the following formula for the *trace* of the semigroup

$$\mathrm{Tr}(e^{-tA}) := \sum_{n=1}^{\infty} e^{-\lambda_n t} = \int_\Omega k_t(x,x)\,\mathrm{d}x.$$

By estimating k_t from above and below for $t \to 0$ and using *Karamata's Theorem*, which relates the behaviour of a positive Borel measure on $[0,\infty)$ near ∞ to that of its Laplace transform near 0, one can recover the Weyl asymptotics. (There is a nice presentation of this in a previous Internet Seminar [A, Lecture 6].)

Another consequence of the min-max principle in mathematical physics is *Courant's theorem* for the number of nodal domains of the eigenfunctions (i.e. connected components of the set where the eigenfunctions are nonzero) of the Laplacian on a bounded domain with Dirichlet (or Neumann) boundary conditions (see [B] or [CH, Ch. VI]). A classical theorem from Sturm–Liouville theory says that the nth eigenfunction of the one-dimensional Laplacian has exactly $n-1$ interior zeros, that is, n nodal domains. Courant's theorem uses the min-max principle and a clever test function argument to show that the nth eigenfunction has *at most* n nodal domains in two or more dimensions.

In fact, one can say more: for only finitely many n can one have equality in Courant's theorem. This refinement due to Pleijel [P] can be shown using the Weyl asymptotics and the *Faber–Krahn inequality*, which gives a lower bound on the first eigenvalue of the Dirichlet Laplacian depending only on the volume of the domain and the dimension.

In this project, we will introduce the above-mentioned principles, the Weyl asymptotics, Courant's theorem and Pleijel's refinement, and sketch their proofs.

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2.3 Project C

Non-autonomous forms: invariance and maximal regularity in H

Luigi Negro (Lecce), Lena Fürnstall (Darmstadt), Fabian Hornung (Karlsruhe), Mohammed Moussi (Oujda)

Coordinators: Dominik Dier and Manfred Sauter (Ulm)

We continue with the theory for non-autonomous equations from Lecture 14, extending it in two directions. On the one hand, we generalise the invariance criteria for convex sets to non-autonomous and inhomogeneous Cauchy problems. In this context we present an application to a semi-linear transport problem

$$\begin{cases} u'(t) - \operatorname{div}\big([a_{kl}(t)]\nabla u\big) = u(t)\big(1 - u(t)\big), \\ u(0) \in \mathcal{C}, \end{cases}$$

where $\mathcal{C} := \{v \in L^2(\Omega) : 0 \leq v(0) \leq 1 \text{ a.e. on } \Omega\}$. We prove that there exists a unique solution u with maximal regularity such that $u(t) \in \mathcal{C}$ for all $t \in [0, T]$.

On the other hand, we study maximal regularity in H, i.e. whether for all $f \in L^2(0, T; H)$ and $u_0 \in V$ the solution $u \in \mathrm{MR}(V, V')$ of

$$\begin{cases} u' + \mathcal{A}u = f, \\ u(0) = u_0 \end{cases}$$

is in $H^1(0, T; H)$, and not merely in $H^1(0, T; V')$. While this is not true in general, we shall prove that if $\mathcal{A}$ is associated with a symmetric, continuous, coercive non-autonomous form that is of *bounded variation*, one does have maximal regularity in H. A form $\mathfrak{a}\colon [0, T] \times V \times V \to \mathbb{K}$ is said to be of bounded variation if there exists an increasing $g\colon [0, T] \to \mathbb{R}$ such that

$$\big|\mathfrak{a}(t, v, w) - \mathfrak{a}(s, v, w)\big| \leq \big(g(t) - g(s)\big) \|v\|_V \|w\|_V$$

for all $s \leq t$ in $[0, T]$ and $v, w \in V$.

Both parts of this project are based on elementary methods and no further prerequisites to the ISEM lectures are required. Our main reference for the project is [Die14]. For the second part a simplified proof will be given that is based on piecewise-autonomous approximation. The results covered in this project are very recent findings that involve topics of active research.

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2.4 Project D

Capacities

Judith Berendsen (Amsterdam), Jan Heckwolf (Darmstadt)

Coordinator: Ralph Chill (Dresden)

The classical Wiener criterion asserts that on a bounded, open subset $\Omega \subseteq \mathbb{R}^n$ the Dirichlet problem

$$\begin{aligned} -\Delta u &= 0 \text{ in } \Omega, \\ u &= g \text{ on } \partial\Omega, \end{aligned} \tag{2.1}$$

admits for every $g \in C(\partial\Omega)$ a unique solution $u \in C^2(\Omega) \cap C(\bar{\Omega})$ if and only if Ω is regular in capacity, that is, for every boundary point $x \in \partial\Omega$ the series

$$\sum_{k=1}^{\infty} 2^{k(n-2)} \operatorname{cap}(\Omega \cap B(x, 2^{-k}))$$

diverges (see (2.37) in [GT]) . Here, the capacity of a set $A \subseteq \mathbb{R}^n$ is defined by

$$\operatorname{cap}(A) := \inf\{\int_\Omega |\nabla u|^2 : u \in H^1 \cap C(\mathbb{R}^n) \text{ and } u \geq 1 \text{ on } A\}.$$

The capacity also appears in the study of regularity of Sobolev functions. For instance, every function $u \in H^1(\mathbb{R}^n)$ admits a representative which is continuous quasi everywhere, that is, up to an exceptional set of capacity 0.

In the literature, the notion of capacity has been generalised in the context of Dirichlet forms on L^2, and the aim of this project is to study some properties of these general capacities. For example, the Poincaré inequality, the logarithmic Sobolev inequality, the Nash inequality can be characterised by conditions involving the capacity, and the above mentioned regularity theorem for Sobolev functions can be generalized to a more general context. For example, the project can be based on Chapter 8 of [BGL] .

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2.5 Project E

Forms and cosine functions

Luciano Abadias (Zaragoza), Ting-Ying Chang (Sydney), Abderrahim Medbouhi (Oujda)

Coordinator: András Bátkai (Budapest and Wuppertal)

Operator cosine functions were invented to help understanding abstract second order Cauchy problems as strongly continuous semigroups are analogous to exponential functions in solving first order (in time) abstract Cauchy problems. There is an excellent introduction into this subject in [ABHN] .

Crouzeix [C] proved that if a form has numerical range in a suitable parabola, then the corresponding operator generates a cosine function. We refer to the monograph by Haase [H] for this result with detailed complete proofs.

In the project we will try to understand this result, and more general connections between numerical range and functional calculus.

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2.6 Project F

Approximation and convergence of forms

Heiko Hoffmann (Karlsruhe), Bas Nieraeth (Utrecht),
Karsten Poddig (Hamburg), Amina Thabet (Gabes)

Coordinator: Tom ter Elst (Auckland)

Let H be a complex Hilbert space. For all $n \in \mathbb{N}$ let a_n be a densely defined sectorial form in H and let A_n be the associated m-sectorial operator. Further, let a be a densely defined sectorial form in H with associated m-sectorial operator A. Suppose that the sequence $(a_n)_{n\in\mathbb{N}}$ of forms converges in some sense to the form a, does it follow that the m-sectorial operators converge to A in some sense? Or that the semigroups converge?

First results for closed positive symmetric forms were proved by Kato and Simon [Kat], [Sim] for convergence to a from above or below. These theorems have been extended to sectorial forms which are not necessarily closed [AE], [BE].

In this project we studied the convergence in a couple of situations, including cases where the forms are not densily defined.

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2.7 Project G

Trotter's Product Formula for Forms

Hai Trinh (Budapest), Khalil Kamal (Marrakesh),
Floris Claassens (Leiden)

Coordinators: E. Ait Benhassi, S. Boulite, L. Maniar (Marrakesh)

The aim of this project is to show the famous Trotter's product formula for a certain class of forms. This formula is known in the semigroup and their generators context. It is well known that this formula is very useful to write the "product" semigroup generated by the sum of two operators (or two forms) in terms of their semigroups. It could be used also to approximate numerically the product semigroup.

To show the Trotter's product formula, we study necessary materials as sectorial and symmetric forms, give their relationship with semigroups. We introduce the notion of resolvent convergence sense, and give its caracterization.

The second part of the project is the Trotter's product formula for projections, as application of the main result. The third part is the application of this fromula to some elleptic operators.

The main references we use are [A], [ABHP], [EN], [K], [S], [Ma].

References

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2.8 Project H

Elliptic operators with first order degeneracy at the boundary

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Coordinator: Roland Schnaubelt (Karlsruhe)

In this project we study the impact of degenerate diffusion coefficients at the boundary of the domain. We establish generation properties and describe the domain of the generator. To avoid rather tedious localization arguments, we focus on model problems on the halfspace $\mathbb{R}^n_+ = \{(x, y) \in \mathbb{R}^{n+1} \,|\, y > 0\}$ and treat in $L^p(\mathbb{R}^{n+1}_+)$, $p \in (1, \infty)$, the differential operator

$$A = -y(\Delta_x + \partial_{yy}) + a \cdot \nabla_x + b\partial_y \tag{2.2}$$

where $a \in \mathbb{R}^n$ and $b \in \mathbb{R}$. The diffusion part degenerates at $\partial\mathbb{R}^{n+1}$ in normal direction of first order, and hence it is 'of the same size' as the drift part.

The behavior of the corresponding elliptic and parabolic problems depends very much on the size and sign of b. The case $b > -1/p$ has been treated in [1] for all $p \in (1, \infty)$. Here A is endowed with the domain

$$D^{\circ}_{p,\mathrm{reg}} = \{u \in W^{1,p}_0(\mathbb{R}^{n+1}_+) \cap W^{2,p}_{\mathrm{loc}}(\mathbb{R}^{n+1}_+) \mid y|D^2u|, \sqrt{y}\,|\nabla u| \in L^p(\mathbb{R}^{n+1}_+)\}.$$

It then generates an analytic semigroup of positive contractions. The domain $D^{\circ}_{p,\mathrm{reg}}$ incorporates a Dirichlet boundary condition and full regularity (each summand of Au belongs to $L^p(\mathbb{R}^{n+1}_+)$ if $u \in D^{\circ}_{p,\mathrm{reg}}$). Replacing here $W^{1,p}_0$ by $W^{1,p}$, we obtain the larger space $D_{p,\mathrm{reg}}$ without boundary conditions.

For $b < -1/p$, one has a strong outward pointing drift term, and rather unexpected phenomena occur. The paper [2] deals with the one-dimensional case (i.e., $A = -y\partial_{yy} + b\partial_y$ and $n = 0$). It is shown that A induces two different generators of positive analytic semigroups if $b \in (-1, -1/p)$: The first one has full regularity, but no boundary conditions; whereas the second one has Dirichlet boundary conditions, but its domain is not contained in $W^{1,p}$. For $b \le -1$, there is just one generator with full regularity and without boundary conditions. For $n \ge 1$, the case $b < -1/p$ is not fully analyzed yet. For $p = 2$, the operator A with the domain $D_{2,\mathrm{reg}}$ generates a positive analytic semigroup on $L^2(\mathbb{R}^{n+1}_+)$. It is contractive for $b \le -1$, but only bounded and not quasi-contrcative if $p \in (-1, -1/2)$, see [3].

The proofs are based on semigroup theory (known from the lectures), approximation arguments, explicit apriori estimates and standard results for partial differential equations with nondegenerate coefficients. The latter ones will be either used without proof or shown by means of form methods.

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2.9 Project I

The Ornstein–Uhlenbeck operator and semigroup in Gauss space.

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Robert Nabiullin (Wuppertal)

Coordinators: Alessandra Lunardi (Parma),
Michele Miranda jr. (Ferrara), Diego Pallara (Lecce)

In this project the aim is to study the Ornstein–Uhlenbeck operator and semigroup in Gaussian spaces. The starting point will be the theory of Dirichlet forms developed in the lectures and the main reference is the first chapter of the book of Bogachev [B].

We start by fixing a Gaussian measure γ on $\mathbb{R}^d$

$$d\gamma(x) = \frac{1}{(2\pi)^{\frac{d}{2}}} e^{-\frac{|x|^2}{2}} dx.$$

Then we consider the Lebesgue and Sobolev spaces $L^2(\gamma)$ and $W^{1,2}(\gamma)$, and we define the Dirichlet form in $W^{1,2}(\gamma)$

$$a(u,v) = \int_{\mathbb{R}^d} \nabla u(x) \cdot \nabla v(x) d\gamma(x)$$

and the associated (Ornstein–Uhlenbeck) operator L that on regular functions is given by

$$Lu(x) = \Delta u(x) - x \cdot \nabla u(x).$$

We shall characterize the domain $D(L)$ of L in $L^2(\gamma)$. The operator $(L, D(L))$ generates a strongly continuous semigroup in $L^2(\gamma)$ which has the following analytical representation

$$T_t u(x) = \int_{\mathbb{R}^d} u(e^{-t}x + \sqrt{1-e^{-2t}}y) d\gamma(y), \qquad \forall u \in L^2(\gamma).$$

Further results that can be presented are the following:

1. spectrum and eigenfunctions (Hermite polynomials) of L and Wiener chaos decomposition of $L^2(\gamma)$;
2. smoothing properties of the semigroup;
3. functional inequalities (Poincaré and Log–Sobolev) by the semigroup approach.

One of the aims of this project is to give a perspective of the subject of 19^{th} Isem which will focus on the extension of this theory to the infinite dimensional setting of abstract Wiener spaces.

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2.10 Project J

The Maz'ja Inequality

Vito Buffa (Ferrara), Sebastian Mildner (Dresden), Christian Schöner (Hagen), Carl Hammann (Dresden)

Coordinators: Daniel Daners (Sydney), Marcus Waurick (Dresden)

The aim of the project is the study of Maz'ja's inequality mentioned in Lecture 12: Let $\Omega \subseteq \mathbb{R}^n$ open, bounded with $\mathcal{H}_{n-1}(\partial\Omega) < \infty$, where $\mathcal{H}_{n-1}$ is the $(n-1)$-dimensional Hausdorff measure. Then there is $c_{\mathrm{M}} > 0$ such that for all $u \in C(\overline{\Omega}) \cap H^1(\Omega)$ we have with $q := \frac{2n}{n-1}$,

$$\|u\|_{L_q(\Omega)}^2 \leq c_{\mathrm{M}} \left(\int_\Omega |\nabla u(x)|^2 \mathrm{d}x + \int_{\partial\Omega} |u(x)|^2 \mathrm{d}x \right).$$

For understanding a proof of this result, among other things, we need to understand Sard's lemma, that is, roughly speaking, the image of critical points of a smooth map of a subset of $\mathbb{R}^n$ with values in $\mathbb{R}^m$ has Lebesgue measure 0. Another important tool is the isoperimetric inequality. The latter asserts that given $n \geq 2$ there exists $c > 0$ such that for all measurable $\Omega \subseteq \mathbb{R}^n$ with finite Lebesgue measure $|\Omega|$, the inequality

$$|\overline{\Omega}|^{\frac{n-1}{n}} \leq c\mathcal{H}_{n-1}(\partial\Omega)$$

holds true.

2.11 Project K

Schrödinger operators with inverse square potentials

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Coordinators: Abdelaziz Rhandi, Cristian Tacelli (Salerno)

Let $A_V = -\Delta - V(\cdot)$ be a Schrödinger operator on $L^2(\mathbb{R}^N)$ with a potential satisfying $0 \leq V \in L^\infty(\{x \in \mathbb{R}^N : |x| \geq \varepsilon\})$ for every $\varepsilon > 0$, and $\lim_{x\to 0} V(x) = \infty$. If V is "too singular" at the origin, can this prevent the existence of positive solution to the heat equation $\partial_t u + A_V u = 0$ (on $\mathbb{R}^N \times (0,\infty)$)?

It has long been known that the corresponding initial value problem is wellposed (in $L^2(\mathbb{R}^N)$ and other spaces) if $V(x) \leq \frac{C}{|x|^{2-\varepsilon}}$ for some positive constants C and ε. H. Brezis and J.L. Lions conjectured that no positive solution will exist if $V(x) \geq \frac{C}{|x|^{2-\varepsilon}}$.

The problem was settled by P. Baras and J.A. Goldstein [1]. Let $C^*(N) = \left(\frac{N-2}{2}\right)^2$ and $V_c(x) = \frac{c}{|x|^2}$ be the inverse square potential with $c \in \mathbb{R}$. Let us consider the Cauchy problem

$$\begin{cases} \partial_t u + A_{V_c} u = 0, & \text{on } \Omega \times (0, \infty), \\ u(x, 0) = f(x), & x \in \Omega, \end{cases} \tag{2.3}$$

with $\Omega = \mathbb{R}^N$ if $N \geq 2$ and for $N = 1$ one has to take for example $\Omega = (0, \infty)$ so as to have a connected spatial domain and add a Dirichlet boundary condition at 0. Baras and Goldstein [1] proved

Theorem 1. *The Cauchy problem (2.3) has a unique positive solution for each* $0 \leq f \in L^2(\Omega)$ *if* $c \leq C^*(N)$ *and no positive solutions at all if* $c > C^*(N)$.

Let $W_n(x) = \inf\{V_c(x), n\}$ be the cutoff potential, with $c > C^*(N)$. Let u_n solve

$$\begin{cases} \partial_t u_n - \Delta u_n - W_n u_n = 0, & \text{on } \mathbb{R}^N \times (0, \infty), \\ u_n(x, 0) = f(x) \geq 0. \end{cases}$$

Here $0 \neq f \in L^2(\mathbb{R}^N)$ or, more generally, f grows no faster than $e^{|x|^{2-\varepsilon}}$ at infinity. Since W_n is bounded, u_n exists. If a positive solution u to (2.3) were to exist, then $0 < u_n \leq u$ which is a contradiction, since $u_n(x, t)$ tends to infinity at all spatial points and at all positive times (see [1, Theorem 2.2.(ii)]). This is called *instantaneous blowup*.

In this project we propose to prove Theorem 1 for $\Omega = \mathbb{R}^N$ with $N \geq 3$ by following the approach of Cabré and Martel [2].

More generally, we consider the Kolmogorov equation perturbed by a singular potential

$$(K_V) \quad \begin{cases} \partial_t u(x,t) = \Delta u(x,t) - \nabla\rho(x) \cdot \nabla u(x,t) + V(x)u(x,t), & t > 0,\, x \in \mathbb{R}^N, \\ u(\cdot, 0) = u_0 \in L^2(\mathbb{R}^N, d\mu). \end{cases}$$

Here we suppose that $0 \leq V \in L^1_{loc}(\mathbb{R}^N)$, $N \geq 3$ and $\rho = 0$ or $\rho \in C^{1+\alpha}_{loc}(\mathbb{R}^N)$, $\alpha \in (0,1)$ is such that $d\mu(x) := e^{-\rho(x)}dx$ is a probability density on $\mathbb{R}^N$. Set

$$\lambda_1(V) := \inf_{0 \neq \varphi \in W_0^{1,2}(\mathbb{R}^N, d\mu)} \left(\frac{\int_{\mathbb{R}^N} |\nabla\varphi|^2\, d\mu - \int_{\mathbb{R}^N} V\varphi^2\, d\mu}{\int_{\mathbb{R}^N} \varphi^2\, d\mu} \right).$$

The Cabré-Martel approach consists in proving the following:

Theorem 2. *Assume the above assumptions. Then the following hold:*

(i) If $\lambda_1(V) > -\infty$, *then there exists a positive weak solution* $u \in C([0, \infty), L^2(\mu))$ *of* (K_V) *satisfying*

$$\|u(t)\|_{L^2(\mu)} \leq M e^{\omega t} \|u_0\|_{L^2(\mu)}, \quad t \geq 0 \tag{2.4}$$

for some constants $M \geq 1$ *and* $\omega \in \mathbb{R}$.

(ii) If $\lambda_1(V) = -\infty$, *then for any* $0 \leq u_0 \in L^2(\mu)$, $u_0 \neq 0$, *there is no positive weak solution of* (K_V) *satisfying (2.4).*

As a consequence and by Hardy's inequality with optimal constant $C^*(N)$:

$$C^*(N)\int_{\mathbb{R}^N}\frac{\varphi^2}{|x|^2}\,dx \le \int_{\mathbb{R}^N}|\nabla\varphi|^2\,dx, \quad \forall\varphi\in C_c^1(\mathbb{R}^N)$$

we obtain Theorem 1.

Another application of Theorem 2 is when $\rho = \frac{1}{2}\langle Bx, x\rangle$, where B a real, symmetric $N\times N$-matrix such that $\sigma(B)\subset(0,\infty)$. Here the use of the following weighted Hardy inequality with optimal constant

$$C^*(N)\int_{\mathbb{R}^N}\frac{\varphi^2}{|x|^2}\,d\mu \le \int_{\mathbb{R}^N}|\nabla\varphi|^2\,d\mu + \|B\|\sqrt{C^*(N)}\int_{\mathbb{R}^N}\varphi^2\,d\mu, \quad \forall\varphi\in C_c^1(\mathbb{R}^N),$$

gives the following result:

Theorem 3. (see [3]) *Assume that $N\ge 3$ and $B=(b_{ij})$ as above. The following assertions hold.*

(i) If $0\le c\le C^*(N)$, then there exists a weak solution $u\in C([0,\infty),L^2(\mu))$ of

$$\begin{cases} \partial_t u(x,t) = \Delta u(x,t) - Bx\cdot\nabla u(x,t) + \frac{c}{|x|^2}u(x,t), & t>0,\ x\in\mathbb{R}^N, \\ u(\cdot,0) = u_0\in L^2(\mu), \end{cases} \tag{2.5}$$

satisfying

$$\|u(t)\|_{L^2(\mu)} \le Me^{\omega t}\|u_0\|_{L^2(\mu)}, \quad t\ge 0 \tag{2.6}$$

for some constants $M\ge 1$ and $\omega\in\mathbb{R}$.

(ii) If $c > C^*(N)$, then for any $0\le u_0\in L^2(\mu)$, $u_0\ne 0$, there is no positive weak solution of (2.5) satisfying (2.6).

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2.12 Project L

Fractional powers and Kato's conjecture

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Coordinators: Moritz Egert, Robert Haller-Dintelmann (Darmstadt)

Let $a : V \times V \to \mathbb{C}$ be a closed, densely defined, sectorial form in a complex Hilbert space H, and let A be the associated sectorial operator. In between the operators $A^0 := \mathrm{Id}$ and $A^1 := A$ a continuous scale of so-called fractional powers of A can be defined via Balakrishnan's formula

$$A^\alpha u := \frac{\sin \pi\alpha}{\pi} \int_0^\infty \lambda^{\alpha-1} A(\lambda + A)^{-1} u \,\mathrm{d}\lambda \qquad (0 < \alpha < 1).$$

Related to these operators is a famous conjecture first formulated by T. Kato in 1961 [K1]:

> *We do not know whether or not* $\mathrm{dom}(A^{1/2}) = \mathrm{dom}(A^{*1/2})$ *[. . .]. This is perhaps not true in general. But the question is open even when A is regularly accretive. In this case it appears reasonable to suppose that both* $\mathrm{dom}(A^{1/2})$ *and* $\mathrm{dom}(A^{*1/2})$ *coincide with* $\mathrm{dom}(a)$*[. . .].*

As a first main result we will consider the so-called second representation theorem[1] to the effect that Kato's conjecture holds provided the sesquilinear form a is symmetric, i.e. A is self-adjoint. We will see that square roots are the borderline case for this type of question: On the one hand

$$\mathrm{dom}(A^\alpha) = \mathrm{dom}(A^{*\alpha}) \qquad (0 \le \alpha < \frac{1}{2}).$$

On the other hand

$$\mathrm{dom}(A^\alpha), \mathrm{dom}(A^{*\alpha}) \subseteq V \qquad (\frac{1}{2} < \alpha \le 1)$$

but for every such α counterexamples can be given for which $\mathrm{dom}(A^\alpha)$ and $\mathrm{dom}(A^{*\alpha})$ are not the same subset of V.

In the critical case $\alpha = \frac{1}{2}$ Kato's conjecture is false for general sectorial operators but it does hold true for differential operators of formal type $Au = -\mathrm{div}(\mu \nabla u)$. This was a long-standing open problem that was eventually solved in 2001 using harmonic analysis' most delicate methods [AHLMT]. These techniques lie far beyond the scope of this

[1] In the terminology of Kato [K2] the first representation theorem is Thm. 12.4 of the lecture notes.

project but we aim to discuss in which sense this result is surprising and how it affects other topics in analysis.

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2.13 Project M

Form inequalities for symmetric contraction semigroups

Elena Golovneva (St. Petersburg), Imen Metoui (Tunis),
Florian Pannasch (Dresden), Marco Peruzzetto (Trento)

Coordinator: Markus Haase (Delft/Kiel)

Let $-A$ be the generator of a strongly continuous semigroup $(S_t)_{t\geq 0}$ of symmetric operators on $\mathrm{L}^1(\mathrm{X})$, where $\mathrm{X} = (X, \Sigma, \mu)$ is an arbitrary measure space. The semigroup is called a *symmetric contraction semigroup* if each operator S_t is L^1- and L^∞-contractive, i.e.

$$\|S_t f\|_1 \leq \|f\|_1 \quad \text{and} \quad \|S_t f\|_\infty \leq \|f\|_\infty$$

for all $f \in \mathrm{L}^1 \cap \mathrm{L}^\infty$. In Lecture 10 of the ISem it has been shown by virtue of complex interpolation techniques that the semigroup $(S_t)_t$ extends to a contractive analytic semigroup on $\mathrm{L}^p(\mathrm{X})$ of angle $\theta_p = \frac{\pi}{2}\Big(1 - |1 - \frac{2}{p}|\Big)$ for $1 < p < \infty$. However, the optimal angle is $\varphi_p = \arccos|1 - \frac{2}{p}|$. In the sub-Markovian case this has been proved first in [LiPe95], see also [Ou05]. The general case is due to Kriegler [Kr11].

The aim of this project is to give a proof based on the recent article [Ha15]. Note that the statement can be equivalently reformulated as

$$\mathrm{Re} \int_{\mathrm{X}} \mathrm{e}^{\pm \mathrm{i}\varphi_p} (Af) \cdot \overline{f} |f|^{p-2} \geq 0 \tag{2.7}$$

for all $f \in \mathrm{dom}(A_p)$, where $-A_p$ is the generator in L^p of the semigroup. In the project we look more generally at inequalities of the form

$$\mathrm{Re} \sum_j \int_{\mathrm{X}} AF_j(\mathbf{f}) \cdot G_j(\mathbf{f}) \geq 0$$

where $\mathbf{f}$ is a $\mathbb{C}^d$-valued measurable function on X and the F_j, G_j are measurable functions on $\mathbb{C}^d$. It is shown that such an inequality holds in general, i.e., for *every* symmetric contraction semigroup, iff it holds for a couple of simple two-dimensional test cases. In this way, (2.7) is reduced to a scalar inequality which can then be established by elementary methods.

The main reduction step involves a clever idea from a recent work of Carbonaro and Dragičević from [CaDr13] and some abstract operator theory, interesting in its own right,

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2.14 Project N

Form methods for evolution equations on graphs

Raphael Gerlach (Padeborn), Julia Kleinhans (Wuppertal),
Salem Nafiri (Marrakesh)

Coordinator(s): Hafida Laasri and Delio Mugnolo (Hagen)

One of the main scopes of this project is to show that already the innocent looking ℓ^2-spaces allow for a rich theory of quadratic forms. The starting point of our project is the classical Dirichlet form introduced in the Lecture 5. One can replace the second derivative operator

$$\Delta : f \mapsto \lim_{h\to 0}\lim_{k\to 0} \frac{f(\cdot + h + k) - f(\cdot + h) - f(\cdot + k) + f(\cdot)}{hk} ,$$

which is well-defined on smooth functions $f : \mathbb{R} \to \mathbb{R}$, by its finite differences counterpart

$$\mathcal{L} : f \mapsto f(\cdot + h) - 2f(\cdot) + f(\cdot - h) .$$

Because $\mathcal{L}f$ is a well-defined object even if f is only defined in countably many points, $\mathcal{L}$ can be considered as a discretisation of the Laplacian.

A similar construction can be actually performed for general functions $f : \mathsf{V} \to \mathbb{R}$, where V is a general (finite or countable) set of points. In order to make sense of the terms like $f(x+h)$, let us turn this unordered set of points into a *graph* by assigning an adjacency structure $\sim$ and replace

$$f(x+h)+f(x-h) \qquad \text{by} \qquad \sum_{\substack{y \in \mathsf{V} \\ y \sim x}} f(y) \ .$$

In this way, differential operators on intervals or domains can be effectively turned, at least formally, into difference operators on graphs – an idea that goes back to Gustav Kirchhoff, who studied the *discrete Laplacian* $\mathcal{L}$ in [Kir47] due to its interplay with the theory of electric circuits. Kirchhoff also found the quadratic form a associated with $\mathcal{L}$; Beurling and Deny then proved in [BD59] that this a is a Dirichlet form (in the sense of Remark 10.14 in the ISem lecture notes) – in fact, the earliest example of Dirichlet form in the literature!

The aim of this project is to present in detail the theory of discrete Laplacians on graphs. We will first introduce in detail the form a in the easy case of finite graphs (the one considered in [BD59]) and re-prove the result by Beurling and Deny, showing that a space-discrete version of the heat equation is governed by a Markovian semigroup. We will then pass to the more delicate case of infinite graphs, which requires the introduction of discrete version h^1 of the common Sobolev space H^1, and show how certain combinatorial properties of the graph influence some functional analytic properties of h^1 following [KL10,Mug13].

Finally, we will consider non-autonomous heat equations on graphs. That is, we will discuss the case of discrete Laplacians $\mathcal{L}$ (or equivalently, quadratic forms a) that are not constant with respect to the time variable: in this part of the project we apply some results on maximal regularity and stability obtained in [ADLO14] and [ADK14] to the case of non-autonomous discrete Laplacian.

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2.15 Project O

Characterisation of L^p-contractivity by invariance criteria

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Marcel Kreuter (Ulm), Marc Wrona (Darmstadt)

Coordinators: Jochen Glück (Ulm), Markus Kunze (Konstanz)

Consider a C_0-semigroup on an L^2-space whose generator is associated to a form a. As we have seen in the lecture, there are abstract results which allow us to check whether a closed convex set C is invariant under the semigroup. This can often be used to prove L^∞-contractivity or L^1-contractivity of the semigroup. In principle, those criteria can also be used to establish L^p-contractivity of the semigroup. This, however, requires knowledge about the projection onto the L^p-unit ball, which, unfortunately, cannot be computed explicitly.

Recently, Nittka overcome this problem by establishing an implicit formula for this projection which contains enough information to apply the abstract invariance criterium. In our talk we present this approach in detail, closely following Nittka's original article [1].

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Monday		Tuesday		Wednesday		Thursday		Friday	
07:30–09:00	Breakfast	07:30–09:00	Breakfast	07:30–09:00	Breakfast	07:30–09:00	Breakfast	07:30–09:00	Breakfast
09:00	Opening	09:00–10:00	**Michel Crouzeix** Functional Calculus based on the Numerical Range	09:00–10:00	**Daniel Lenz** Intrinsic metrics for Dirichlet forms	09:00–10:00	**Ralph Chill** Gradient systems associated with j-elliptic functionals	09:00–10:30	**Project F** Tom ter Elst
09:15–10:15	**Tom ter Elst** On convergence of Dirichlet-to-Neumann operators		Coffee Break		Coffee Break		Coffee Break		
	Coffee Break	10:30–12:00	**Project M** Markus Haase	10:30–11:30	**Project A** Peer Christian Kunstmann	10:30–12:00	**Project E** András Bátkai		Coffee Break
10:45–12:15	**Project J** Marcus Waurick			11:30–12:30	**Project D** Ralph Chill		Conference Photo	11:00–12:30	**Project N** Hafida Laasri, Delio Mugnolo
12:30	Lunch	12:30	Lunch	12:30	Lunch	12:30	Lunch	12:30	Lunch
14:30–16:00	**Project C** Dominik Dier, Manfred Sauter	14:30–16:00	**Project G** El Mustapha Ait Ben Hassi, Said Boulite, Lahcen Maniar	14:30	Excursion	14:30–16:00	**Project H** Roland Schnaubelt	14:30–16:00	**Project K** Abdelaziz Rhandi, Cristian Tacelli
	Coffee Break		Coffee Break				Coffee Break		Coffee Break
16:30–18:00	**Project B** James Kennedy	16:30–18:00	**Project O** Jochen Glück, Markus Kunze			16:30–18:00	**Project I** Alessandra Lunardi, Michele Miranda, Diego Pallara	16:30–18:00	**Project L** Moritz Egert, Robert Haller-Dintelmann
18:30–19:30	Dinner	18:30–19:30	Dinner	18:30–19:30	Dinner	18:30–19:30	Conference Dinner	18:30–19:30	Dinner

19:30 Coordinators Meeting