GAMM S23: Programm & Abstracts

March 15-19, 2021 - Kassel, Germany

Programm

Wednesday 17.03.21:

Session Chair: Birgit Jacob

16:30-17:10	**R. Schnaubelt** L^p spectrum of Ornstein Uhlenbeck operators
17:10-17:50	**B. Augner** The fast-sorption–fast-surface-reaction limit of a heterogeneous catalysis model
17:50-18:30	**D. T. Kołaczek** Split-operator algorithm for the Moyal equation. Comparative studies of second and fourth order factorizations

Thursday 18.03.21:

Session Chair: Hafida Laasri:

08:30-09:10	**I. Veselic** Three applications of scale uniform uncertainty relations for Schrödinger equations
09:10-09:50	**A. Hussein** Analysis of the primitive equations with horizontal viscosity
09:50-10:30	**B. Haak** Recent progress in Functional Calculus

Session Chair: Ivan Veselic:

14:00-14:40	**A. Radl** Embeddability of matrices into real and positive semigroups
09:10-09:50	**J. Kaiser** Riesz bases of port-Hamiltonian systems
15:20-16:00	**L. Vorberg** Pseudospectrum enclosures by discretization

Friday 19.03.21:

Session Chair: Roland Schnaubelt:

08:30-09:10	**N. Skrepek** Well-posedness of linear first order port-Hamiltonian Systems on multidimensional spatial domains
14:40-15:20	**C. Wyss** Stability under structured perturbations
09:50-10:30	**M. Kurula** Approximate robust output regulation of boundary control systems

Abstracts

Roland Schnaubelt (Karlsruhe Institute of Technology, Germany)

We show that the spectrum of hypoelliptic Ornstein-Uhlenbeck operators on $L^p(\mathbb{R}^n)$ are the sum of the spectra of the diffusion and the drift operators. The spectrum of the diffusion part is the negative halfline and that of the drift term is either a (possibly p-dependent) line parallel to the imaginary axis or a discrete subgroup of $i\mathbb{R}$. The proof is based on spectral theory of semigroups and scaling arguments.

This is joint work with Simona Fornaro (Pavia), Giorgio Metafune and Diego Pallara (Lecce).

Björn Augner (Technische Universität Darmstadt, Germany)

Every mathematical model describing physical phenomena is an approximation to model reality, hence has its limitations. Depending on characteristic values of the variables in the model, different aspects of the model and, e.g., thermodynamic mechanisms have to be emphasised, or may be neglected in a reduced limit model. Within this talk, a heterogeneous catalysis system will be considered consisting of a bulk phase (chemical reactor) and an active surface (catalytic surface), between which chemical substances are exchanged via adsorption (transport of mass from the bulk boundary layer adjacent to the surface, leading to surface-accumulation by a transformation into an adsorbed form) and desorption (vice versa). Quite typically, as is the purpose of catalysis, chemical reactions on the surface occur several orders of magnitude faster than, say, chemical reactions within the bulk, and sorption processes are often quite fast as well. Starting from the non-dimensional version, different limit models, especially for fast surface chemistry and fast sorption at the surface, are considered. For a particular model problem, questions of local-in-time existence of strong and classical solutions, positivity of solutions and blow-up criteria for global existence are addressed.

Damian Tadeusz Kołaczek (AGH University of Science and Technology, Poland)

In the most widely used formulation of non-relativistic quantum mechanics of isolated systems, pure states of the system are described by complex-valued wave functions belonging to the Hilbert space of square-integrable functions. In turn, observables are represented by self-adjoint operators acting on that space. Alternatively non-relativistic quantum mechanics can be also formulated in the phase space of position and momentum variables which are treated on equal footing. Due to the position-momentum uncertainty principle it is not possible to define legit probability density distribution over the phase space. However, we can define a class of non-classical distribution functions that contain all information about the state of the system. One of them is the Wigner Distribution Function (WDF). This function is real-valued, bounded, and has proper marginal distributions, but it can take negative values in some regions of the phase space. This feature of the WDF can be understood as a signature of nonclassical effects. In this formulation dynamical variables are defined by the Weyl symbols of closed densely defined operators acting on the Hilbert space of the square-integrable functions. Time evolution of the WDF is given by the Moyal equation. For this equation we can define a unitary time evolution operator which belongs to strongly continuous one-parameter group generated by the Moyal bracket acting on the WDF. The time evolution operator can be approximately factorized into the product of operators such that each of them can be diagonalized by adequate Fourier transform, which allows very fast and efficient computations.
We used two approximations, i.e. second and fourth order factorization methods, to compare their performance in computations of the time evolution of several isolated systems. The approximation error for small enough values of time step becomes acceptable and larger evolution times can be realised by successive applications of the time evolution operator with smaller time step due to the group property of that operator [1].

[1] D. Kołaczek, B.J. Spisak, and M. Wołoszyn "The phase-space approach to time evolution of quantum states in confined systems: the spectral split-operator method", Int. J. Appl. Math. Comput. Sci. 29, 439 (2019).

Ivan Veselic (Technische Universität Dortmund, Germany)

I will present recent results on quantitative unique continuation estimates for functions in appropriately chosen subspaces which lead to uncertainty relations and so-called spectral inequalities, respectively. The mentioned appropriately chosen subspaces could be defined in terms of the properties of the Fourier transform or as spectral subspaces of an elliptic second order operator. The problems which I consider are defined on the whole Euclidean domain or on large boxes, which may be considered as an approximation of the whole space. The obtained estimates are uniform over the family of such geometries.
Three applications will be considered: (1) Shifting estimates for eigenvalues, including ones in gaps of the essential spectrum, as well as shifting estimates for edges of components of essential spectrum, under the influence of a semidefinite potential (2) Anderson localization for general classes of random potentials with small support, and (3) null-controllability of the heat equation with interior control.

Amru Hussein (Technische Universität Kaiserslautern, Germany)

The 3D-primitive equations are one of the fundamental models for geophysicalflows, and they are used for describing oceanic and atmospheric dynamics. They are derived from Navier-Stokes equations assuming a hydrostatic balance. Here, the initial value and time-periodic problem for the primitive equations with only horizontal viscosity is studied. The motivation to study this problem is that in many geophysical models the horizontal viscosity is considered to be dominant and the vertical viscosity is neglected.
From the analytical point of view such models with only partial viscosity terms are also very interesting since they combine features of both parabolic diffusion equations in horizontal directions and hyperbolic transport equations in vertical direction. Roughly speaking one thus expects that regularity is preserved in vertical direction while it is smoothed in horizontal directions. Following this intuition allows us to identify classes of initial data for which this problem is locally or even globally well-posed.
This is based on a joint work with Martin Saal and Marc Wrona.

Bernhard Haak (University of Bordeaux, France)

In my talk I will select a subject that reflect recent developments in functional calculi.

Agnes Radl (Technische Universität Berlin, Germany)

It is a well-known problem whether a Markov matrix is embeddable into a Markov semigroup, see the recent survey [1]. We consider a similar problem: Given a (finite or infinite) real/positive matrix T, is it embeddable into a real positive matrix semigroup, i.e., is there a real/positive matrix semigroup $(T(t))_{t\geq 0}$ such that $T(1) =$T? We will give necessary and sufficient conditions for embeddability.

The presentation is based on a joint work with Tanja Eisner [2].

[1] M. Baake and J. Sumner, "Notes on Markov embedding", to appear in Linear Algebra Appl.
[2] T. Eisner and A. Radl, "Embeddability of real and positive matrices", preprint.

Julia Kaiser (Bergische Universität Wuppertal, Germany)

Riesz bases are a generalization of orthonormal bases. More precisely, a sequence of vectors in a Hilbert space H is a Riesz basis for H if there exists a boundedly invertible operator S such that the image of the sequence under the operator S is an orthonormal basis of H. A sequence of closed subspaces in a Hilbert space H is a Riesz basis of subspaces of H if there exists an boundedly invertible operator S such that the image of the sequence of subspaces is a complete system of pairwise orthogonal subspaces of H. An operator is called a Riesz operator if there is a sequence of eigenvectors or a sequence of spectral projections which is a Riesz basis (of subspaces) of H.

We consider a special kind of hyperbolic partial differential equations on a one dimensional spatial domain. In particular, we are interested in partial differential equations which can be formulated using a port-Hamiltonian operator of order 1. This class includes wave equations, for example.
In this talk we show that a port-Hamiltonian operator which generates a strongly continuous semigroup a is a Riesz operator if and only if the port-Hamiltonian operator is the generator of a strongly continuous group.

Lukas Vorberg (Bergische Universität Wuppertal, Germany)

Determining the spectrum of a linear operator is a major task in analysis and numerics. The explicit computation of the whole spectrum by analytical or numerical techniques is possible only in rare cases. Moreover the spectrum is in general quite sensitive with respect to small perturbations of the operator. One is therefore interested in supersets of the spectrum that are easier to compute and also robust under perturbations. A well-studied superset of the spectrum is the pseudospectrum. Using finite element discretization of elliptic partial differential operators, we obtain new exclusion regions for the pseudospectrum of the original differential operator.

Nathanael Skrepek (Bergische Universität Wuppertal, Germany)

Port-Hamiltonian systems are physically motivated systems that conserve or at least do not multiply a certain quantity, the Hamiltonian, which is usually the energy of the system. Moreover, interconnections of those systems are again port-Hamiltonian systems. In particular we will consider such systems that have multidimensional spatial dependencies. Hence, we cover the 2D and 3D wave equations, the Maxwell equations, and the Mindline plate model. We want to justify existence and uniqueness of solutions. We approach this by associating a boundary triple to this system. Hence, we can characterize all boundary conditions that provide unique solutions that are non-increasing in the Hamiltonian. As a by-product we develop the theory of quasi Gelfand triple. The presented framework harmonizes existing theories for the wave equation and the Maxwell equations and provides justification for the Mindlin plate model.

Christian Wyss (Bergische Universität Wuppertal, Germany)

Let A be the generator of an exponentially stable C_0-semigroup. Suppose that A is subject to structured perturbations of the form $A^\Delta = A + B\Delta C$ with fixed linear operators B, C and an unknown (or uncertain) bounded operator Δ. The stability radius is the largest bound $r \geq 0$ such that exponential stability is preserved for all Δ with norm less than r. It was introduced by Hinrichsen and Pritchard in 1986 in the finite-dimensional setting and then generalised into various directions. We derive formulas and estimates for the stability radius for infinite-dimensional interconnected systems, in which the influence of Δ is restricted by a prescribed connection structure. Our results are new even in the finite-dimensional case.

Mikael Kurula (Abo Akademi University, Finland)

We extend the internal model principle to systems with boundary control and boundary observation, and construct a robust controller for this class of systems. As a consequence of the internal model principle, any robust controller for a plant with infinite-dimensional output space necessarily has infinite-dimensional state space. We proceed to formulate the approximate robust output regulation problem and present a finite-dimensional controller structure to solve it.
Our main motivating example is a wave equation on a bounded multidimensional spatial domain with force control and velocity observation at the boundary. In order to illustrate the theoretical results, we construct an approximate robust controller for the wave equation on an annular domain and demonstrate its performance with numerical simulations.