GAMM 2022, Section S 23: Applied Operator Theory

Program and Abstracts

August 15–19, 2022, Aachen, Germany

Program

Thursday, August 18, 2022:

Session Chair: Christian Seifert

17:30–18:10 **Karsten Kruse**
On the Lumer-Phillips theorem for bi-continuous semigroups

18:10–18:30 **Jan Meichsner**
On Positivity and Contractivity of Semigroups generated by Fractional Powers

Friday, August 19, 2022:

Session Chair: Jan Meichsner

08:30–09:10 **Jonathan Rohleder**
A new approach to the hot spots conjecture

09:10–09:30 **Timo Reis**
An operator-theoretic approach to port-Hamiltonian systems

09:30–09:50 **Katharina Klioba**
Approximation of Random Evolution Equations

09:50–10:10 **Amru Hussein**
Non-self-adjoint boundary conditions on graphs

10:10–10:30 **Christian Seifert**
Final State Observability Estimates in Banach Spaces

Abstracts

Amru Hussein (Technische Universität Kaiserslautern, Germany)

Classical boundary conditions for elliptic operators include for instance Dirichlet, Neumann and Robin boundary conditions. Going from scalar valued to vector valued functions raises the complexity of the problem considerably. An early pathbreaking approach in this direction has been developed at the begining of the 20th century by the classical Birkhoff–Tamarkin theory for ordinary differential operators. A key issue is how to parametrize boundary conditions and how to ensure basic spectral properties. Here, I consider the model case of a Laplacian on a finite metric graph subject to general non-self-adjoint match- ing conditions imposed at the graph's vertices. A regularity criterium related to the Cayley transform of boundary conditions is discussed and spectral properties of such operators are investigated, in particular similarity transforms to self-adjoint operators and the generation of C_0-semigroups. Concrete examples are discussed exhibiting that non-self-adjoint boundary conditions can yield to unexpected spectral features. How this can be transfered to the situation of partial differential operators on domains is outlined.
The talk is based on joint works with David Krejcirik (Czech Technical Uni- versity in Prague), Petr Siegl (Queen's University Belfast) and Delio Mugnolo (FernUniversität Hagen).

Katharina Klioba (Technische Universität Hamburg, Germany)

Solving random evolution equations numerically requires a discretisation in space, in time, and of random parameters. Methods to treat these three problems separately are well-known, including rates of convergence. In this talk, conditions are presented under which these rates of convergence are conserved for the fully discretised solution. Focusing on spatial discretisation, a quantified version of the Trotter-Kato theorem corresponding to the weak formulation is presented. On a semigroup level, this corresponds to approximating form-induced semigroups on separable Hilbert spaces by restricting the form to simpler, often finite-dimensional, approximating spaces. Rates of strong convergence are obtained on dense subspaces under a joint condition on properties of both the form and the approximating spaces. As a standard application, results are discussed for the heat equation with random coefficients.

Karsten Kruse (Technische Universität Hamburg, Germany)

The well-known Lumer-–Phillips theorem characterises the generators of norm-strongly continuous contraction semigroups on Banach spaces. Namely, a closed and densely defined operator $(A, D(A))$ on a Banach space generates a norm-strongly continuous contraction semigroup if and only if $(A, D(A))$ is dissipative and $\lambda - A$ is surjective for some $\lambda > 0$.
However, in many applications of semigroups of operators on Banach spaces the semigroups are not norm-strongly continuous but strongly continuous with respect to a weaker Hausdorff locally convex topology τ. Examples of such semigroups are adjoint semigroups of norm-strongly continuous semigroups, implemented semigroups, and transition and Koopman semigroups on the space of bounded continuous functions on a Polish space.

These examples belong to the general framework of bi-continuous semigroups which were introduced by Kühnemund. The Lumer–Phillips theorem was recently extended to a subclass of bi-continuous semigroups by Budde and Wegner, using a Lumer–Phillips theorem for equicontinuous semigroups on Hausdorff locally convex spaces by Albanese and Jornet. The result of Budde and Wegner has a significant limitation since it is only applicable to operators that generate τ-equicontinuous semigroups (after rescaling). However, even a classical bi-continuous semigroup like the Gauß–Weierstraß semigroup on the space of bounded continuous functions is not τ-equicontinuous (even after rescaling) where τ is the compact-open topology. This limitation comes from a concept of dissipativity that is not suitable for bi-continuous semigroups. In this talk we show how to overcome this limitation by using a dissipativity concept w.r.t. to the so-called mixed topology.
The talk is based on a joint work with Christian Seifert.

Jan Meichsner (Technische Universität Dresden, Germany)

Generators of semigroups are in particular sectorial operators and admit therefore fractional powers which, under certain circumstances, are again generators. If a given semigroup is positive and/or contractive, one may ask whether these properties get inherited to the semigroups generated by the fractional powers. The talk will give simple sufficient conditions when this is the case and demonstrate them at the example of the Laplacian on domains and graphs.

Timo Reis (Technische Universität Ilmenau, Germany)

The port-Hamiltonian way of modelling results in a class of systems which has a special form, and this class is moreover closed under interconnection. Moreover, many partial differential equations have a structure which — at least formally — resembles the one from ordinary port-Hamiltonian systems. In the talk we are looking for a suitable formulation of infinite dimensional port-Hamiltonian systems which cover various examples from mechanics and electrodynamics. In particular, our class includes systems with boundary control and observation. Our approach is based on the formulation of infinite dimensional systems via so-called "system nodes" from Staffans [1].

[1] O. Staffans: Well-Posed Linear Systems, Cambridge University Press, 2005

Jonathan Rohleder (Stockholm University, Sweden)

It is a conjecture going back to J. Rauch (1974) that the hottest and coldest spots in an insulated homogeneous medium such as an insulated plate of metal should converge to the boundary, for "most" initial heat distributions, as time tends to infinity. This so-called hot spots conjecture can be phrased alternatively as follows: the eigenfunction(s) corresponding to the first non-zero eigenvalue of the Neumann Laplacian on a Euclidean domain should take its maximum and minimum on the boundary only. This has been proven to be false for certain domains with holes, but it was shown to hold for several classes of simply connected or convex planar domains. One of the most recent advances is the proof for all triangles given by Judge and Mondal (Annals of Math. 2020). The conjecture remains open in general for simply connected or at least convex domains. In this talk we provide a new approach to the conjecture. It is based on a non-standard variational principle for the eigenvalues of the Neumann and Dirichlet Laplacians.

Christian Seifert (Technische Universität Hamburg, Germany)

Given a semigroup $(S_t)_{tgeq0}$ on a Banach space X, a bounded linear operator $C\colon X \to Y$ for another Banach space Y and $T > 0$ we study final state observability estimates, i.e. existence of $C_{\mathrm{obs}} \geq 0$ such that for all $x \in X$ we have

$$\|S_T x\|_X leq C_{\mathrm{obs}} \left(\int_0^T \|CS_t x\|_Y^r \,\mathrm{d}t \right)^{1/r},$$

where $r \in [1, \infty)$ (and suitably modified for $r = \infty$). Such estimates stem from investigating systems

$$\begin{aligned} \dot{x}(t) &= -Ax(t) \quad (t > 0), \\ y(t) &= Cx(t) \quad (t > 0), \end{aligned}$$

where $-A$ generates $(S_t)_{t\geq 0}$. We will review recent results on final state observability estimates in Banach spaces and show an application for semigroups coming from parabolic equations in L_p-spaces. This is based on joint works with C. Bombach (Chemnitz), F. Gabel (Hamburg), D. Gallaun (Hamburg) and M. Tautenhahn (Leipzig).