Efficient Calculation of Fluid Structure Interaction in Ship Vibration

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Abstract

Because of its high mass density, water has a significant impact on vibrations of ship structures and fluid-structure interaction has to be considered in an accurate modeling of global ship vibration. In this presentation we discuss an approach for including the effect of water in such simulations efficiently.

1 Modeling ship vibration.

Forces induced by engines and propellers excite ship vibrations which despite of their tiny amplitudes (of approximately 10^{-1} mm) can affect human comfort and may cause fatigue damages. Predicting ship vibrations it is indispensable to account for the effect of the surrounding water because the hydrodynamic forces acting on the ship's hull can considerably reduce the natural frequencies of the dry ship.

The hydrodynamic influence of the water on the vibrations of a ship is modeled as added mass. The displacement of the structure causes the fluid near the interface to move which in turn exerts an opposing force on the ship's hull. The additional force is needed to accelerate the surrounding fluid which by Newton's law can be interpreted as an additional mass of the structure called hydrodynamic mass or added mass.

Since the structural displacements are very small compared to the dimensions of the ship, the governing equations for the ship and the water can be assumed to be linear. Moreover the flow of the water around the ship's hull can be supposed to be inviscid and irrotational. Hence, the velocity field of the fluid is the gradient of a volicity potential ϕ which due to mass conservation satisfies the Laplace equation $\Delta\phi = 0$ in the fluid domain.

Assuming an infinitely wide and deep fluid domain, boundary conditions have to be specified at the free water surface and at the submerged ship's hull. The exact boundary condition at the free surface is nonlinear (cf. [5]), but for frequencies above 1 Hz and small displacements it can be linearized to yield a pressure release condition which takes the form $\phi = 0$. At the submerged ship's surface the fluid velocity normal to this surface must be equal to the normal velocity of the structure,

$$\frac{\partial \phi}{\partial n} = u \cdot n, \tag{1.1}$$

where u denotes the velocity of the structural displacement and n is the outward normal.

To account for the surrounding water a FE model of the ship can be complemented by a FE discretization of the Laplace equation with coupling boundary condition (1.1) (cf. [1]). This causes considerable additional cost since only a bounded part of the water can be modeled this way and suitable boundary conditions on the outer boundary have to be specified or the remaining unbounded region of the water has to be discretized by semiinfinite elements. If boundary element (BE) methods are used in combination with a special fundamental solution, an unbounded fluid domain can be modeled, but only the finite surface of the ship's hull has to be discretized. Thus the problems previously mentioned are avoided. The mesh can simply be generated from the FE mesh of the ship's hull. Today this is a standard approach if three dimensional effects have to be included in the analysis (cf. [3]).

An advantage of this method is the fact that the discretization of the coupled fluid-solid problem has the same dimension as the FE model of the dry ship alone in case the free surface boundary condition is handled using the method of images [7]. However, as a drawback the part of the mass matrix corresponding to the wet hull of the ship is fully populated.

Hence, the resulting eigenvalue problem governing ship vibrations has the following form:

$$K_S x = \lambda(M_S + M_H)x \tag{1.2}$$

where $K_S, M_S \in \mathcal{R}^{n\times n}$ are the stiffness and mass matrix of a FE model of the ship which are large-scaled and sparse. The hydrodynamic mass matrix M_H models the impact of the surrounding water on the ship. Only rows and columns of M_H corresponding to wet degrees of freedom contain entries which are different from zero. But this is still quite a large number n_H, and discretizing the potential of the velocity field of

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the water by a BE method this part of M_H becomes fully populated. Moreover, its explicit form is usually not known, but only matrix-vector products $M_H x$ are accessible via the (approximate) solution of the linear BE system.

2 A reduction method

Solving the eigenvalue problem (1.2) by the shift-and-invert Lanczos method [2] one has to solve a linear system

$$(K_S - \sigma(M_S + M_H))x = b \tag{2.3}$$

for x in every iteration step. This can be done by a direct method which requires the explicit form of the matrix M_H (i.e. n_H solves of the BE system) and which due to the structure of the system matrix $A := K_S - \sigma(M_S + M_H)$ is very time and memory consuming. Solving (2.3) by an iterative solver like MINRES [6] every Lanczos step requires a suitable Krylov subspace. Hence, one has to apply the system matrix A to a couple of vectors, and each of these multiplications demands the solution of one BE system.

In the following we derive a reduction method which causes much less cost. The hydrodynamic mass matrix M_H is symmetric and positive definite. We take advantage of the spectral decomposition

$$M_H = \sum_{i=1}^{n_H} \lambda_i x_i x_i^T \tag{2.4}$$

of M_H where the eigenvalues $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{n_H}$ are ordered by magnitude. Then the best approximation to M_H by a matrix of rank $\tilde{n}$ with respect to the spectral norm is the truncation of (2.4)

$$\tilde{M}_H = \sum_{i=1}^{\tilde{n}} \lambda_i x_i x_i^T. \tag{2.5}$$

The Lanczos method is favorable for computing an approximation to $\tilde{M}_H$ since it does not employ the explicit form of the matrix M_H but only matrix-vector products and according to the Kaniel–Paige theorem [4] it converges first to extreme eigenvalues and in particular to the largest ones which are better separated than the smallest ones. Notice that the Lanczos process for computing $\tilde{M}_H$ can even be accelerated replacing the solution of the BE approximation by the fast multipole approach [7].

Having determined the approximation $\tilde{M}_H$ to M_H we have to solve the reduced eigenvalue problem

$$K_S x = \lambda(M_S + \tilde{M}_H)x. \tag{2.6}$$

Tackling it by the shift-and-invert Lanczos method one has solve a linear system

$$(K_S - \sigma(M_s + \tilde{M}_H))x = b \tag{2.7}$$

in every iteration step where σ is a preselected shift.

Since K_S and M_S are sparse the LU factorization of $K_S - \sigma M_S$ can be determined efficiently and since the rank of $\tilde{M}_H$ is quite small compared to the dimension of the problem it is inexpensive to employ the Sherman–Morrison–Woodbury formula [4] for solving problem (2.7).

With this approach the shift-and-invert Lanczos method for the reduced problem (2.6) essentially requires the following work: To initialize one provides those vectors and matrices which are independent of the right hand side b when solving (2.7) by the Sherman-Morrison Woodbury formula. To this end a slightly larger number than $\tilde{n}$ solves of the BE system are necessary in a Lanczos process for computing $\tilde{M}_H$, and $\tilde{n}$ solves of linear systems $(K_S - \sigma M_S)w_j = x_j$ and $\tilde{n}^2$ scalar products of length n are required in this preprocessing phase. Thereafter every iteration step requires one solve of a linear system of dimension n, and $\tilde{n}$ scalar products of length n.

We evaluated our approach for a FE model of a typical container vessel of 252.5 m length and 32.2 m breadth with $n = 35262$ degrees of freedom, $n_H = 2032$ of which are contained in the submerged part of the hull. Details about the accuracy and run-times of our reduction method will be contained in the full paper.

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