

Use of Automatic Differentiation for Sensitivity Analysis of Flight Loads

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Abstract

This paper addresses the use of sensitivities for flight loads analysis in pre-design with deeper consideration of gradients. A possible way of obtaining them from an implementation of the equations of motion by means of automatic differentiation and the embedding in an available framework will be discussed. An example with parametrized inaccuracies in the aerodynamic lift distribution shows that the use of automatic differentiation is a feasible and promising approach.

1 INTRODUCTION

Basic scope of loads computation in aircraft engineering is to prove airworthiness according to Part 25 in the FAR/JAR. In practise the design process is performed in so called loops, in which it has to be shown with increasing accuracy under only minor design changes that the structure withstands all possible loads. Major design changes at that stage are extremely expensive and time consuming.

Hence the goal is to fix so called target loads, which are not to be exceeded, already in early stages of the development even though not all inputs are available with final accuracy. For example windtunnel tests underlie long development cycles while many parameters and even the configuration of new materials like CFRP for some components may not be finally decided at that point of time. To be able to provide reliable input data for the loop process, knowing the effect of uncertain design parameters on loads is essential. In addition sensitivities can be useful for optimization in pre-design processes.

In Section 2 we will present the model equations. We show how parameters can be introduced and how uncertainties can be described. An implementation into an existing framework will be discussed in Section 3 with results of a numerical simulation presented in Section 4.

2 LOADS COMPUTATION AND UNCERTAINTIES

In this work our scope is on loads computation by finding trim solutions, which can be expressed as root finding problems. The most simple case is horizontal, steady flight. However, a yawing or a roll manoeuvre can be trimmed as well as a 2.5g vertical balanced manoeuvre using multiple trim steps.

2.1 Equations of Motion

The mathematical model contains the equations of motion along with a description of the external forces on the aircraft. In this paper we use a version based on a work by Waszak et. al., as derived in [10], which combines a non-linear set of rigid body equations with a linear set of flexible body equations. The equations are described in [6] as follows:

$$M_{b1b1}\dot{V}_{b1} - V_{b1} \times \Omega_{b2} - T_{b1e}g = \Phi_{b1a}P_a^{ext} \quad (1)$$

$$I_{b2b2}\dot{\Omega}_{b2} + \Omega_{b2} \times (I_{b2b2}\Omega_{b2}) = \Phi_{b2a}P_a^{ext} \quad (2)$$

$$M_{ff}\ddot{u}_f + D_f\dot{u}_f + K_f u_f = \Phi_{fa}P_a^{ext} \quad (3)$$

Here the first two equations denote the non-linear part with V_{b1} the velocity in three dimensions, Ω_{b1} the according rotational velocities and the mass and inertia matrices M_{b1b1} and I_{b2b2} . Further, u_f denotes the deflection of the flexible mode shapes, M_{ff} , D_{ff} and K_{ff} refer to flexible the mass-, damping- and stiffness matrix. T_{b1e} is a transformation matrix from geodatic to body and the Φ_{xa} denote the transformation of the external forces into the available degrees of freedom.

The most engineering challenge is in the determination of the external forces P_a^{ext} , which themselves are a function of the states of the aircraft and additional variables. In our model we consider aerodynamic effects due to steady flight, e.g. V_{b1} and Ω_{b2} , control surface deflections u_{x2} and flexibility, e.g. u_f and \dot{u}_f in addition to a very simple engine model with thrust T_n . More sophisticated models including unsteady effects, turbulence or gyroscopic effects are possible. The equations can easily be extended introducing the Euler angles and a position vector.

To obtain a trim solution, the second order differential equation is rewritten into a first order one using

$$\tilde{u}_f = \begin{pmatrix} u \\ \dot{u} \end{pmatrix}, \quad (4)$$

doubling the size of the third equation. The whole equation system is rewritten as root finding problem for the stationary system (1)-(3) and solved approximately by a Newton-type method.

However, the number of the resulting equations now is smaller than the number of unknowns. Hence as many entries as necessary from V_{b1} , \dot{V}_{b1} , Ω_{b2} , $\dot{\Omega}_{b2}$, \ddot{u}_f , $\dot{\ddot{u}}_f$, u_{x2} and T_n are fixed. For example to obtain level flight $V_{b1}(3)$, \dot{V}_{b1} , Ω_{b2} , $\dot{\Omega}_{b2}$ and \ddot{u}_f are set to zero, $V_{b1}(1)$ is set to a cruise speed, while for example the thrust, ailerons, rudder and HTP are left free.

In practise it can be observed, that neither the uniqueness nor the existence of a solution to such a problem is guaranteed. However, missing convergence in such a case usually denotes a situation that physically does not make sense.

From the solutions afterwards the local loads

$$P_a = P_a^{ext} - \begin{bmatrix} M_{b1b1}(T_{b1e}g + \Omega_{b2} \times V_{b1} + \dot{V}_{b1}) \\ I_{b2b2}\dot{\Omega}_{b2} \\ M_{ff}\ddot{u}_f \end{bmatrix} \quad (5)$$

can be recovered and integrated to the points, where their value is of interest.

2.2 Parametrized Data

A parametrized design makes sense as soon as there are distinct patterns, how systems are built up. For example modern commercial aircraft in general look very similar, as they are mainly constructed from wing-like and fuselage-like components, which can be parametrized as described in [2]. Patterns occur at other inputs than structure as well. A simple approximation of the lift distribution can be derived from the surface of a parametrized wing, e.g. with a vortex lattice method as described in [5].

There are even patterns in the typical errors in a lift distribution, that can be observed during flight tests. For example the efficiency of a control surface is often underestimated or some components show larger stiffnesses in reality than in the FE models assumed.

In that sense the choice of parameters gives a link between engineering knowledge and the underlying numerical model. A new observation can be formed into a pattern, described by a choice of values. These values may in addition be connected by equations or inequalities, e.g. to exclude unfeasible combinations.

2.3 Ways of Describing Uncertainties

An exact and intuitive way of describing uncertainties is to define intervals in which parameters may vary. By propagating these intervals through the equations of motion it is possible to obtain exact limits for the loads at the end of the calculation. The only drawback is that these intervals may become very large and include combinations of parameters with very small probability, which cannot be distinguished from common events. A large amount of parameters in combination with realistic uncertainty bandwidths may soon sum up to intervals which overestimate the actual risk heavily.

For decisions concerning the design typically stochastic statements are more appropriate. Knowing the probability for a possible maximum load exceeding the envelope is small, it might make sense to neglect it, especially if there are possible ways to compensate rare exceedences. But good probability distributions propagated through

the equations of motion may be very difficult to obtain. A possible solution, which could be applied to trim solutions, is described in [3].

Often sufficient for engineering decisions are gradients S of loads P_a due to a parameter p :

$$S = \frac{\partial P_a}{\partial p}. \quad (6)$$

Since experience shows that most trim problems are well behaving, a gradient gives a good first impression of the stability of the solution, e.g. of the dependency on a parameter.

One way of obtaining the gradients is to use automatic differentiation, which is extensively done in scientific computing, see [4]. Several automatic differentiation codes are already available in many languages. They generate a derivative code, which can be compiled and computes gradients or Jacobian matrices.

The basic idea is that it does neither use divided differences nor symbolic differentiation but a distinct procedure to analyse functions and rewrite them using the chain rule. The resulting code can be compiled and computes derivatives that are exact down to machine precision usually faster than using divided differences. Interestingly, the derivative code includes the whole numerical method, which may be useful to investigate regions where the method becomes unstable.

3 VARLOADS ENVIRONMENT

VarLOADS, described in [6], is a program for loads calculation written for the MATLAB/SIMULINK environment and intended to be a flexible tool for pre-design and special investigations. Its most important feature is its modular approach. Each effect, described as a component of the model, can be treated and exchanged separately from an extendable library. Even the core of the whole program, the equations of motion (1)-(3), can be exchanged easily by more complex, coupled mathematical models as proposed by [9].

The languages used to describe and solve or integrate the equations can as well be varied, as long as all interfaces stay the same. Standard is a SIMULINK model, solved by an inexact Newton's method or the trimroutine TRIMEX written at the DLR. But a pure MATLAB-solution, which is used here, does exist as well. More sophisticated languages could be introduced easily.

3.1 Extension with automatic differentiation

The VarLOADS framework can be extended by a suitable modul for the sensitivity analysis. Using the MATLAB version of the code that solves the equations of motion for a trim solution, we have used the tool ADIMAT [1] that generates a MATLAB derivative code. The resulting code forms a new module and along with loads the derivatives with respect to introduced parameters can be delivered.

4 EXAMPLE

In this paper we consider a simple academic example using one single parameter only for clarity of exposition. The steady equations of motion (1)-(3) are approximated by an inexact Newton's method. Load recovery (5) is applied.

We will differentiate with respect to a parameter p using [8], which is scaling the distribution of the aerodynamic lift on wing as proposed by T. Kier [7] such that with increasing p the lift is decreased on the outer side and it is increased on the inner side. The parameter p as well has effect on the lift gradient with respect to the angle of attack α and on the center of pressure such that it will influence the solution of the trim problem.

The investigation was performed for a 1.3g manoeuvre trimmed by the HTP only, but an extension to other manoeuvres is possible. As an example a typical long range passenger aircraft is used with a vortex lattice method for obtaining a basic lift distribution before it is modified with p .

The code includes the solution of the equations of motion, load recovery and integration to the so called output stations, at which they are needed to reconstruct the design. Altogether this involves 18 m-files containing about 600 lines of executable code. The example is constructed using one parameter of interest only and having one result vector containing more than 700 elements of which the gradient is to be computed automatically. Intermediate results include gradients of vectors with more than 3000 elements. Note, however, that the evaluation of gradients with respect to multiple parameters is also possible.

Since not all matlab features are yet fully supported, some rather extended modifications on the original code were necessary at the expense of performance. The derivative code then had to be adapted slightly only.

4.1 Visualization

Basically the visualization of the results of sensitivity analysis can be handled like the visualization of loads itself, shown for our example in Figure 1. Over the relative wing span the relative gradient is plotted not only for the exact gradient obtained by automatic differentiation but as well for gradients approximated by divided differences using several differences h .

Figure 1-a shows the sensitivity of the integrated shear force Q_z , for which the parameter shows quite a significant influence near the center of the wing but little influence at the root. That is not too surprising, since the overall lift, which carries the aircraft, cannot change very much near the wing root. In contrast M_x shows significant sensitivity there as denoted in Figure 1-b.

In this case divided differences mostly underestimate the gradient. Figure 2 shows the dependence of the error for the root wing bending moment on the size of the difference in p as percentage of the exact gradient. If the difference is too large, the gradient changes over the difference and thus causes an error. If the difference is too small, rounding errors come into effect.

Figure 3 shows another way of getting an overview on the global effects on the

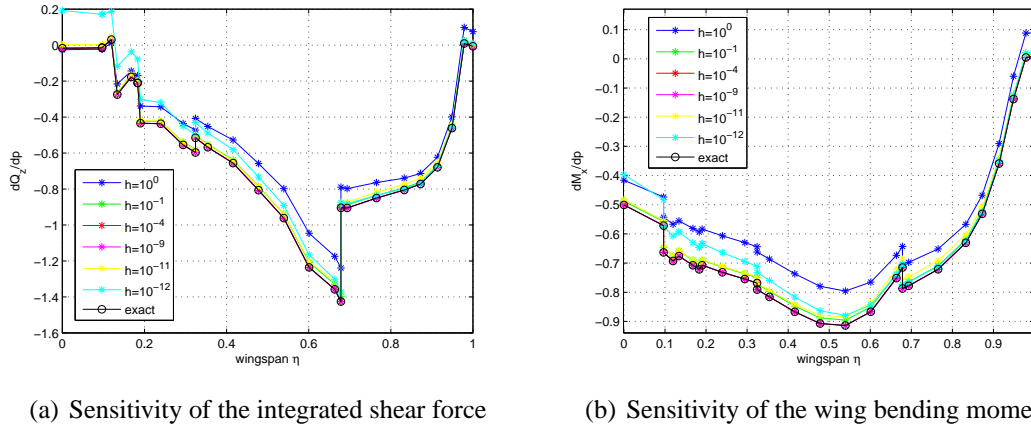


Figure 1: Relative gradients plotted over the wing span. The sensitivity of Q_z near the wing root is almost zero. As expected M_x decreases with p near the wing root.

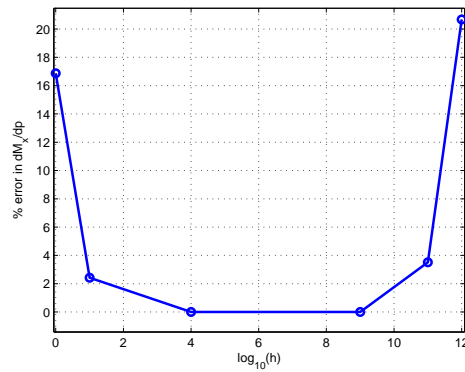


Figure 2: The error made by divided differences using a delta h with respect to the exact gradient. The scale on the abscissa is logarithmic.

aircraft. The gradient $\frac{\partial Q_z}{\partial p}$ is plotted as a circle with a radius proportional to the absolute magnitude for every output value. It can be seen that the change in the flight attitude causes a different shear force on the horizontal tail plane which is propagated through the fuselage to the wings.

Basically these plots could be drawn for multiple parameters as well. But it should be noted, that parameters do sum up and hence always the worst case would be shown independently of their probability. At this point assumptions on the probability of each gradient could help, such that instead of a circle with fixed radius one fading with increasing radius would be drawn, such that greyscales correspond to events with diminishing probability. This is one goal of our future research.

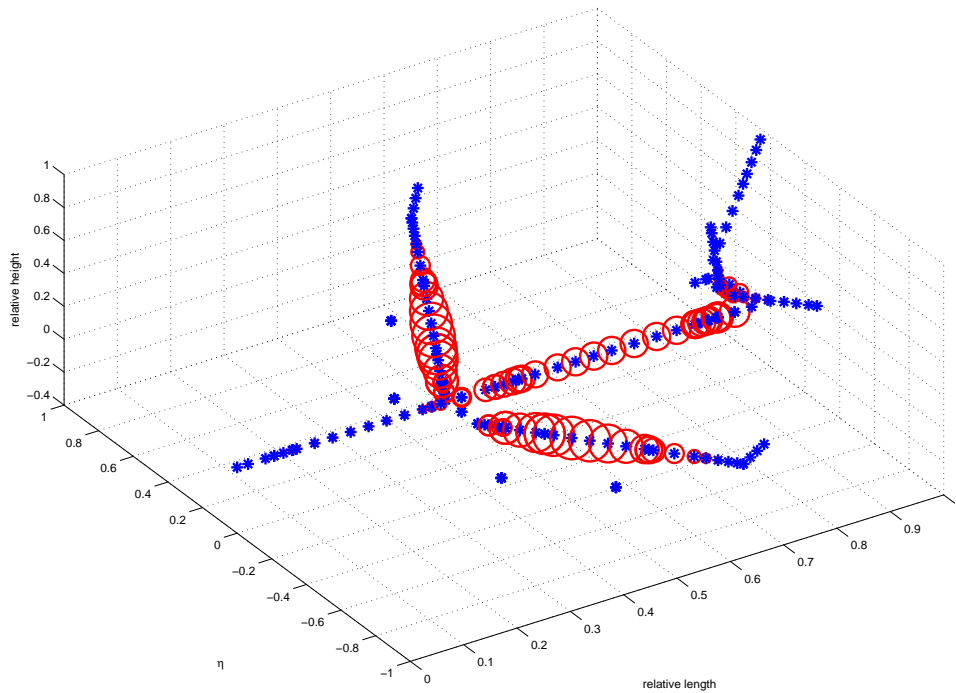


Figure 3: The magnitude of the gradient of Q_z for every output station drawn as a circle with according radius for the whole aircraft.

5 CONCLUSION AND FUTURE WORK

In this work we have discussed the need for sensitivities and possible ways of obtaining them. Ideas how sensitivities, i.e. gradients with respect to corresponding parameters, can be generated and implemented into an existing framework for loads calculation were presented. Preliminary results of an experiment show the feasibility of obtaining sensitivities in the sense of gradients by the tools of automatic differentiation.

It was shown that the results obtained by divided differences may depend strongly on the size of the used difference and can hence lead to misleading results. But even though it sounds charming to have a derivative exact down to machine precision for some applications it might still be useful to work with divided differences, since they reflect the behaviour of loads due to a parameter in the vicinity of the solution as well. Still a comparison with an exact derivative may validate the used difference.

For more than a few parameters the discussion about visualization suggests that stochastic methods are inevitable to give sensible measures of risks due to the collection of parameters as a whole. Here as well the tools of automatic differentiation can give useful approaches as described in [3].

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