Discontinuous Galerkin Spectral Element Methods - Space-Time Formulations and Efficient Solvers

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Abstract

We are interested in constructing cheap and efficient implicit high order solvers for compressible turbulent flow problems. These problems arise for example in the design of next generation jet engines, air frames, wind turbines or star formation. A suitable high order discretization for these problems are discontinuous Galerkin spectral element methods (DG-SEM). In this talk we discuss challenges of solvers for DG-SEM discretizations in space combined with implicit time-stepping methods.

One option to yield implicit DG-SEM solvers is to apply a space-time DG-SEM discretization, i.e. discretizing space and time simultaneously with DG-SEM. We present two approaches for the formulation and implementation of space-time DG-SEM: Either time is treated as an additional coordinate direction and the Galerkin procedure is applied to the entire problem. Alternatively, the method of lines is used with DG-SEM in space and the fully implicit Runge-Kutta method Lobatto IIIC in time. The two approaches are mathematically equivalent in the sense that they lead to the same discrete solution. However, in practice they differ in several important respects, including the terminology used to the describe them, the structure of the resulting software, and the interaction with nonlinear solvers. We present challenges and merits of the two approaches and show their impact on numerical tests using implementations based on the Distributed and Unified Numerics Environment (DUNE).

Another option to construct implicit DG-SEM solvers is the classical method of lines approach. The spatial directions are discretized with DG-SEM and any implicit time-stepping method can be applied to the resulting ODE. This yields large nonlinear systems and a solver has to be chosen carefully. We suggest to use a preconditioned Jacobian-free Newton-Krylov method. The challenge here is to construct a preconditioner without constructing the Jacobian of the spatial discretization. Our idea is to make use of a simplified replacement operator for the DG operator and a multigrid method. We discuss the idea of our suggested preconditioner and present numerical results to show the potential of this preconditioning technique.