

LEAKY CONICAL SURFACES: SPECTRAL ASYMPTOTICS, ISOPERIMETRIC PROPERTIES, AND BEYOND

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The study of spectra of differential operators on conical domains is motivated by the desire to characterise spectral properties of differential operators on more general unbounded domains with asymptotically flat boundary. As a part of this program, we considered in [BEL14] three-dimensional Schrödinger operator

$$H_{\mathcal{C}} = -\Delta - \alpha\delta(\cdot - \mathcal{C})$$

with attractive δ -interaction of strength $\alpha > 0$ supported on the circular conical surface $\mathcal{C} = \{(x, y, z) \in \mathbb{R}^3 : z = \cot(\theta)\sqrt{x^2 + y^2}\}$ with $\theta \in (0, \pi/2)$. The Hamiltonian $H_{\mathcal{C}}$ can be defined as a self-adjoint operator in $L^2(\mathbb{R}^3)$ via its quadratic form. In quantum mechanics this Hamiltonian models a charged particle attracted by the (leaky) surface \mathcal{C} . We showed in [BEL14] that $\sigma_{\text{ess}}(H_{\mathcal{C}}) = [-\alpha^2/4, +\infty)$ and that the discrete spectrum of $H_{\mathcal{C}}$ is infinite.

In this talk, we are going to present recent results on the asymptotics of the eigenvalue counting function $\mathcal{N}_{-\alpha^2/4-E}$ for $H_{\mathcal{C}}$. We obtained in [LO16] that $\mathcal{N}_{-\alpha^2/4-E}$ behaves as

$$\mathcal{N}_{-\alpha^2/4-E} \sim \frac{\cot \theta}{4\pi} |\ln E|, \quad E \rightarrow 0 + .$$

We intend also to discuss spectral properties of Schrödinger operators with δ -interactions supported on more general (not necessarily circular) cones. In this context, we proved in [EL15] that under certain reasonable constraints the circular cone maximises the lowest eigenvalue. In addition, we plan to briefly highlight several related questions, one of them concerns a generalization of this spectral problem to higher space dimensions and another one is about spectral properties of δ -interactions supported on truncated cones.

These results are obtained in collaboration with J. Behrndt, P. Exner, and T. Ourmières-Bonafos.

REFERENCES

- [BEL14] J. Behrndt, P. Exner, and V. Lotoreichik, Schrödinger operators with δ -interactions supported on conical surfaces, *J. Phys. A* **47** (2014), 355202.
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- [LO16] V. Lotoreichik and T. Ourmières-Bonafos, On the bound states of Schrödinger operators with δ -interactions on conical surfaces, *to appear in Comm. Partial Differential Equations*, *arXiv:1510.05623*.