Verified solution of ODEs by Taylor models implemented in MATLAB/INTLAB

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Solving differential equations rigorously is a main and vigorous topic in the field of verified computation. Here, solving rigorously means that a computer program supplies an approximate solution along with error bounds that respect all numerical as well as all rounding errors that occurred during the computation. An exact solution is proved to be enclosed within these rigorous bounds.

In this context so-called Taylor models have been used successfully for solving ordinary differential equations (ODEs) rigorously. Implementations are COSY INFINITY [1], FLOW [2], ODEIntegretor [3], and RIOT [4]. Here, COSY INFINITY developed by Berz and Makino and their group is the most advanced implementation. Recently, we implemented the Taylor model approach in MATLAB/INTLAB [5].

We give a short introduction to Taylor models, their rigorous arithmetic, and the Taylor model method for enclosing solutions of ordinary differential equations in a verified manner. We only treat initial value problems

$$y' = f(t, y), \qquad y(t_0) = y_0$$

where the initial value y_0 may be an interval vector. For specific ODEs we demonstrate how to use and call our verified ODE solver. This is designed to be very similar to calling MATLAB's non-verified ODE solvers like ode45. Finally, results and run times are compared to those of COSY INFINITY, RIOT and Lohner's classical AWA.

References

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